Hemodynamic model for analysis of Doppler ultrasound indexes of umbilical blood flow

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Kleiner-Assaf, Ayala, Ariel J. Jaffa, and David Elad. Hemodynamic model for analysis of Doppler ultrasound indexes of umbilical blood flow. Am. J. Physiol. 276 (Heart Circ. Physiol. 45): H2204–H2214, 1999.—A hemodynamic model for pulsatile fluid flow in a pressurized thin-walled elastic tube was applied for the computation of volumetric blood flow and velocity profiles for a given set of system parameters at any selected location along the umbilical artery. The velocity profiles over one heart cycle provide the fetal blood flow velocity waveforms (FVW) from which the usual Doppler indexes (DI) can be derived. The model was used for a comprehensive investigation of the correlation between DI and system parameters that reflect the anatomy and physiology of umbilical blood flow. The simulations showed that the radial location of the Doppler measurement is insignificant for the calculated DI, whereas the axial site is important. The analysis showed that decreasing the diameter or increasing the length of the umbilical artery reduces fetal mean blood flow rate and increases the DI. Increasing blood viscosity tends to induce similar patterns, whereas decreasing arterial compliance or increasing blood density decreases the DI with little effect on blood flow rate. Fetal heart rate has a minor effect on both DI and fetal blood flow rate. This study provides insight into the dependence of DI on the anatomic and physiological characteristics of umbilical blood flow.

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local deformations in both longitudinal (z) and circumferential (θ) directions.

Blood flow in the umbilical artery is assumed to be an axisymmetric pulsatile flow of an incompressible Newtonian liquid and is modeled by superposition of a small time-dependent component and a component of steady flow. The steady flow component is driven by a linear pressure gradient between the umbilicus and the placenta and is modeled as a steady laminar flow in an elastic tube (10). The unsteady component is modeled as wave propagation through a viscous, incompressible fluid contained in a pressurized elastic tube (4). As in previous works (3, 15, 17), this approach was taken to ensure a mean blood flow over the whole cardiac cycle. The oscillating solutions were obtained by linearization of the governing equations (as explained in Appendix B), and thus summation of the steady and unsteady components is justified.

Steady flow in an elastic tube. The steady component of the flow through an elastic tube is assumed to be laminar (parabolic distribution) and to have a local resistance as in Poiseuille’s flow (10). The radius of the elastic tube R₀(z) changes with the tube mean internal pressure P₀(z), which varies along the z axis. Accordingly, the pressure-flow relationship can be described in a way similar to laminar flow by

\[
\frac{dP₀(z)}{dz} = - \frac{8\mu}{\pi R₀(z)^4} Q_s
\]

where Q_s is the steady volumetric flow rate and μ is the fluid viscosity.

The stresses within the tube wall that are caused by the mean internal pressure are given by

\[
T_{\text{iii}} = \frac{P₀(z) - R₀(z)}{h} \quad T_{\text{zz}} = \frac{P₀(z) - R₀(z)}{2h} \quad T_{\text{rr}} = P₀(z)
\]

where T_{iii}, T_{zz}, and T_{rr} are the internal stresses in the circumferential, longitudinal, and radial directions, respectively, and h is the tube wall thickness. The circumferential strain (ε_{θθ}) for a linear material is given by

\[
ε_{θθ} = \frac{R₀(z) - R₀}{R₀} = \frac{P₀(z)}{R₀} - 1
\]

where R₀ is the tube radius in its unstressed state (P₀ = 0). The constitutive equation for an isotropic elastic material provides the strain-stress relationship

\[
ε_{θθ} = \frac{1}{E} (T_{\text{iii}} - \sigma T_{\text{zz}} - \sigma T_{\text{rr}})
\]

where E is Young's modulus and σ is Poisson's ratio.

Equations 2-4 can be manipulated to give an expression for R₀(z) in terms of P₀(z), the tube geometry, and material properties. Substitution in Eq. 1 and integration between the umbilicus and the placenta yields

\[
Q_s = - \frac{\pi}{8\mu L} \int_0^{P₀(U)} \frac{R₀(z) - R₀}{(R₀(z) - 1)^4} dP₀
\]

where P₀,U and P₀,P are the mean arterial pressures in the placenta and the umbilicus, respectively, and L is the tube length.

The steady area-averaged velocity at any axial location z is given by

\[
\mathbf{W}(z) = \frac{Q_s}{\pi R₀(z)^2}
\]

and the steady parabolic velocity profile is

\[
W₀(r, z) = W_{\text{max}}(z) \left[ 1 - \left( \frac{r}{R₀(z)} \right)^2 \right]
\]

where W_{max} = 2W₀ is the maximal velocity at the center line (r = 0).

Pulsatile flow in an elastic tube. The unsteady component of the pulsatile flow is assumed to be induced by propagation of small waves in a pressurized elastic tube. The mathematical approach is based on the classic model for the fluid-structure interaction problem, which describes the dynamic equilibrium between the fluid and the thin tube wall (4, 31, 32). The dynamic equilibrium is expressed by the hydrodynamic equations (Navier-Stokes) for the incompressible fluid flow and the equations of motion for an elastic tube, which are coupled together by the boundary conditions (BC) at the fluid-wall interface. The motion of the liquid is described in a fixed laboratory coordinate system, and the dynamic equilibrium of a tube element in its deformed state is expressed in a Lagrangian (material) coordinate system, which is attached to the surface of the tube (Fig. 2).

Fig. 2. Mechanics of arterial wall. A: axisymmetric wall deformation. B: element of tube wall under biaxial loading. R₀, elastic tube radius; \( \hat{r}, \hat{\theta}, \hat{z} \), fixed system coordinates; \( \hat{n}, \hat{t}, \hat{\theta} \), Lagrangian coordinates; T, internal stress.
A brief summary of the mathematical formulation is given in Appendix B. Following Atabek and Lew (4), the first-order approximations for the fluid velocity radial \( u_r \) and axial \( w_z \) components and the pressure \( p_z \) as a function of time \( t \) and space \( r, z \) are given by

\[
u_r(r, z, t) = \frac{A_1 \beta}{\rho_F} \left[ \frac{r}{R_0} + m \frac{J_0(\alpha_0 R_0 z)}{J_0(\alpha_0)} \right] \exp[i \omega(t - z/c)]
\]

\[
w_z(r, z, t) = \frac{A_1}{\rho_F} \left[ 1 + m \frac{r}{R_0} \right] \exp[i \omega(t - z/c)]
\]

\[
p_z(r, z, t) = A_1 \exp[i \omega(t - z/c)]
\]

where

\[
m = \frac{2 + x[2\sigma - (1 - \tau_0)]}{x[(1 - \tau_0)F_{10} - 2\sigma]}
\]

\[
x = \frac{E h}{(1 - \sigma^2)R_0 \rho_F c^2}
\]

\[
c = \frac{2 \cdot c_0}{((k + 2) + [(k + 2)^2 - 8k(1 - \sigma^2)]^{1/2})}
\]

\[
\tau_0 = T_h/1 - \sigma^2
\]

\[
\alpha = \omega R_0/c
\]

\[
\beta = -\omega R_0/c
\]

The time-dependent umbilical blood flow rate at a given point \( z \) between the umbilicus and the placenta is

\[
Q(z, t) = \frac{A_1}{\rho_F} \cdot \pi R_0^2(1 + m \cdot F_{10}) \exp[i \omega(t - z/c)]
\]

\[
-\frac{\pi}{8\mu L} \cdot \int_0^{\rho_p} \frac{R_{oo}(z) - R_{oo}}{E h} Z_{oo}^2(1 - \sigma/2) \cdot P_0(z) - 1 \, dp_0
\]

The umbilical cord normally consists of two umbilical arteries. Accordingly, the mean umbilical flow rate at a given point \( z \) in one period is

\[
Q_{mean}(z) = \frac{2}{T} \int_0^T Q(t, z) \, dt
\]

where \( T \) is the time of one heartbeat.

Computational technique. The longitudinal blood velocity \( W \) at any location within the umbilical artery was computed from Eq. 13 for a given set of independent system parameters and an assumed pressure waveform within the tube. Following Womersley (31, 32), we assumed a harmonic wave for the oscillatory component of the pressure (see Eq. B15), which is given in Eq. 10 for the first harmonic that represents the heart rate. A more realistic blood pressure wave is composed of \( k \) harmonics and can be described by the following Fourier series

\[
p(z, t) = \sum_{n=1}^{k} A_n \exp[i \omega(t - z/c)]
\]

where \( A_n \) is a complex number and is given by an amplitude \( (M_{n,i}) \) and phase shift \( (\varphi_n) \)

\[
A_n = M_n \exp(i \varphi_n) = M_n(\cos \varphi_n + i \sin \varphi_n)
\]

In the absence of data from direct measurement of the pressure waveform in umbilical arteries of humans or experimental animals, we used here the first six harmonics listed in Table 1, which were derived from pressure measurements in the pulmonary artery of a dog (5). This choice may be justified by the fact that the pulmonary arterial pressure of the fetus is the same as its systemic arterial pressure (20), and the systolic and diastolic values of this waveform are similar to those of umbilical blood pressure (70 and 45 mmHg, respectively).
The variation of the velocity with time at any fixed location within the tube yields the FVW for this location. The envelope FVW that is usually measured by commercial Doppler systems is obtained from the maximal axial velocity in the vessel’s cross section at each time step. The DI were computed from the FVW according to Eqs. A1–A3. The system parameters that specify the geometry and material properties of the umbilical artery and those of blood characteristics and pressure are detailed in Table 2. The reference values were taken from the literature to describe an averaged normal umbilical artery. A range of values was chosen to study the contribution of selected anatomic and physiological factors to umbilical blood flow and the corresponding DI. Variations of a single parameter were carried out while the rest of the parameters were kept at the reference values.

RESULTS

The model directly provided the axial velocity at any given time and location within the tube for a specified anatomic and physiological variable of umbilical blood flow, and this allowed the derivation of FVW at selected measurement locations. Figure 3 depicts envelope FVW for a normal sample and shows the reference parameters together with the variations for extreme values (Table 2) of the system parameters. The DI for the reference set of parameters at the umbilicus end \((z = 0)\) were \(PI = 1.00\), \(RI = 0.62\), and \(S/D = 2.62\), and the mean umbilical blood flow \((Q_{\text{mean}})\) was 92 ml/min.

The present model enabled the analysis of the dependence of DI on the location of Doppler ultrasound measurements. Accordingly, the DI were evaluated from FVW computed for the set of reference parameters at 10 different radial locations \((r^* = r/R_0 = 0–0.9)\) on the fetal umbilicus. Similarly, the DI were also computed at 11 equally spaced locations \((z^* = z/L = 0–1.0)\) along the umbilical artery (from umbilicus to placenta). The results are shown in Fig. 4. The DI are almost independent of radial location for most of the cross section, whereas pulsatility decreased (up to 50%) toward the placenta.

### Table 1. Fourier components of input pressure wave

<table>
<thead>
<tr>
<th>Harmonic</th>
<th>(M_n, \text{mmHg})</th>
<th>(\phi_n, ^\circ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8.97</td>
<td>147</td>
</tr>
<tr>
<td>2</td>
<td>2.43</td>
<td>265</td>
</tr>
<tr>
<td>3</td>
<td>0.72</td>
<td>261</td>
</tr>
<tr>
<td>4</td>
<td>0.81</td>
<td>348</td>
</tr>
<tr>
<td>5</td>
<td>0.17</td>
<td>19</td>
</tr>
<tr>
<td>6</td>
<td>0.56</td>
<td>57</td>
</tr>
</tbody>
</table>

Values were derived from measurements in dog pulmonary artery (5).

### Table 2. Parameters used in this work

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Reference Value</th>
<th>Examined Range</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>FHR, beats/min</td>
<td>140</td>
<td>80–240</td>
<td>20</td>
</tr>
<tr>
<td>(P_{0,P}, \text{mmHg})</td>
<td>25</td>
<td>5–45</td>
<td>25</td>
</tr>
<tr>
<td>(P_{\text{max}}, \text{mmHg})</td>
<td>60</td>
<td>52–96</td>
<td>5</td>
</tr>
<tr>
<td>(P_{0,U}, \text{mmHg})</td>
<td>50</td>
<td>15–75</td>
<td>20</td>
</tr>
<tr>
<td>(R_0, \text{mm})</td>
<td>2</td>
<td>1–4.4</td>
<td>25</td>
</tr>
<tr>
<td>(L, \text{cm})</td>
<td>40</td>
<td>20–150</td>
<td>21</td>
</tr>
<tr>
<td>(h/R_0)</td>
<td>0.15</td>
<td>0.05–0.25</td>
<td>26</td>
</tr>
<tr>
<td>(\mu, \text{Poise})</td>
<td>0.04</td>
<td>0.01–0.08</td>
<td>31</td>
</tr>
<tr>
<td>(\rho_f, \rho_w, \text{kg/m}^3)</td>
<td>1.050</td>
<td>500–2,150</td>
<td>31</td>
</tr>
<tr>
<td>(\sigma)</td>
<td>0.5</td>
<td>0.05–0.5</td>
<td>6</td>
</tr>
<tr>
<td>(E, \text{Pa})</td>
<td>0.5</td>
<td>0.1–0.8</td>
<td>6</td>
</tr>
</tbody>
</table>

FHR, fetal heart rate; \(P_{0,P}\), mean arterial pressure at placental insert; \(P_{\text{max}}\), maximal unsteady pressure; \(P_{0,U}\), mean arterial pressure at umbilicus insert; \(R_0\), artery radius; \(L\), artery length; \(h/R_0\), wall thickness-to-radius ratio; \(\mu\), blood viscosity; \(\rho_f, \rho_w\), blood and wall density; \(\sigma\), Poisson ratio (wall); \(E\), Young’s modulus.
The role of arterial anatomy in the determination of DI was examined by computing FVW for a range of arterial diameters ($d = 1.0$–$4.4$ mm), lengths ($L = 20$–$150$ cm), wall thicknesses ($h/R_0 = 0.05$–$0.25$), and elastic moduli ($E = 1$–$8 \times 10^5$ Pa). The computed DI for arterial diameter and length are shown in Fig. 5 as are the corresponding $Q_{\text{mean}}$. The vertical lines (indicated by “Ref”) represent the results for the reference values given in Table 2. Generally, the DI decreased as the artery diameter increased or its length decreased. $Q_{\text{mean}}$ increased with increases in the arterial diameter and decreased as the arterial length increased. In an artery with increased wall thickness the DI decreased, but $Q_{\text{mean}}$ was almost unaffected (Fig. 6). The role of increasing Young’s modulus ($E$) of the artery is mechanically equivalent to that of thickening its wall thickness; thus the variability of DI and $Q_{\text{mean}}$ is similar to that of wall thickness (Fig. 6).

The contribution of blood properties to the pattern of DI and the corresponding blood flow rate was investigated by varying blood density ($\rho = 500$–$2,150$ kg/m$^3$) and viscosity ($\mu = 0.01$–$0.08$ Poise). The results are depicted in Fig. 7 and show that DI decreased as blood density increased or viscosity decreased. The mean blood flow rate was independent of blood density but decreased as blood viscosity increased.

The influence of physiological parameters was studied by varying blood pressure and FHR. Figure 8 shows that FHR had no effect on either DI or the mean blood flow rate. To examine the contribution of peak input pressure (steady – unsteady), we varied the reference value of $P_{\text{max}}$ by multiplying the fundamental harmonic of the unsteady pressure to yield a range of values around the reference value ($P_{\text{max}} = 52$–$96$ mmHg) with
negligible changes in the mean arterial pressure (55–54 mmHg). The resulting DI increased with \( P_{\text{max}} \), whereas \( Q_{\text{mean}} \) was almost unaffected (Fig. 8).

Variation of the mean arterial pressure at the placental insert (\( P_{0,P} \)) in the range of 5–45 mmHg with maintenance of the mean arterial pressure at the umbilicus insert (\( P_{0,U} \)) constant at 50 mmHg is shown in Fig. 9. As downstream pressure increased, \( Q_{\text{mean}} \) decreased and the DI increased. Variation of \( P_{0,U} \) in the range of 15–75 mmHg while \( P_{0,P} \) was held constant at 10 mmHg resulted in peak pressures of 25–85 mmHg. The corresponding DI decreased as \( P_{0,U} \) increased, whereas \( Q_{\text{mean}} \) increased (Fig. 9). Another simulation was conducted by changing \( P_{0,U} \) and \( P_{0,P} \) in such a way that the ratio \( P_{0,P}/P_{0,U} = 0.5 \) was kept constant. This yielded results that are similar to the simulations with \( P_{0,P} \) held constant at 10 mmHg (Fig. 9).

The large volume of computed values of DI and \( Q_{\text{mean}} \) for a range of selected anatomic and physiological parameters (Table 2) allowed investigation of the correlation between the blood flow rate and the DI. For this purpose we plotted \( Q_{\text{mean}} \) versus each of the DI for six system parameters that were found to affect the measured DI (Fig. 10). The diameter of the umbilical artery was found to be the most influential parameter, whereas peak umbilical pressure had the least effect.

The FVW is strongly dependent on the spectral content of the input pressure wave. Attempts were made to simulate an abnormal FVW with a postsystolic notch, a condition that is known to be representative of high-resistance circulation and to have good correlation to poor obstetrical outcome (22). For this purpose, the second and fourth harmonics were multiplied by 2 and the third harmonic by 3. The resulting pressure wave and FVW at three locations along the artery in comparison with the corresponding waves in normal conditions are shown in Fig. 11. It can be seen that the postsystolic notch depth becomes smaller at locations closer to the placenta. The DI for the dicrotic notch condition measured at \( z = 0 \) (umbilicus) were PI = 1.34, RI = 0.8, and S/D = 4.88, values that are very close to those in normal conditions.

**DISCUSSION**

A mathematical model of blood flow in the umbilical artery was used to examine the variability of DI with respect to anatomic and physiological parameters that dictate fetal blood flow. Knowing the blood flow field at any axial and radial location of the umbilical artery provided the FVW and enabled a detailed investigation of the spatial variability of DI. In addition, a direct correlation between the estimated DI and umbilical blood flow rate could be obtained from the model.

**Measurement site.** Medical ultrasound machines including Doppler instruments usually measure the envelope FVW for calculations of DI. In this study, we investigated the variability of DI that were derived at different radial locations (Fig. 4) and from the envelope FVW. The difference between DI calculated at different radial locations (but away from the arterial wall) are
relatively small and may be of the order of magnitude of the system resolution. The DI calculated from the envelope FVW were almost identical to those measured at the FVW from the centerline. Hence, the exact radial site of measurement does not affect the accuracy of the obtained DI. On the other hand, the longitudinal site of Doppler measurements greatly affects the FVW and, as a result, the DI as well (Fig. 4). Accordingly, it is useful to conduct measurements at a defined location to ensure reproducibility and provide data that can be used for comparison. Because only the umbilicus (the fetal side) and the placental insert of the umbilical artery can be identified with certainty, it is now largely recommended to measure DI of the umbilical artery on the placental site (1).

Anatomic factors. The diameter and the length of the umbilical artery have a strong effect on the estimated DI. The results showed that either long umbilical arteries or small diameters yielded increased values of DI (Fig. 5), which can be explained by the increased resistance to blood flow as one would expect in a steady laminar flow according to Poiseuille law. The prediction of DI for small-diameter arteries can be related to pathologies and abnormalities such as arterial occlusions or the presence of a single artery. Pernoll (21) reported that 14% of infants with a single umbilical artery will die perinatally and >50% of them will have structural defects. The decrease of DI as the gestational age increases toward the end of pregnancy (2) may be explained by the increase of the umbilical artery diameter.

The length of the umbilical cord at term is determined by the amount of amniotic fluid present during the first and second trimesters and by the fetal movement. If oligohydramnios (reduced amount of amniotic fluid), amniotic bands, or limitation of fetal motion occur for any reason, the umbilical cord will not develop to an average length (21). A too-short cord may cause...
premature separation of the placenta, especially during parturition. A long umbilical cord can twist around the fetus and create problems such as cord compression and true knot of the cord, which disturb blood flow and in extreme cases may completely block blood flow. The model predicted low flow velocities for long umbilical arteries but did not take into account complications such as cord compression, although the influences on the flow are similar in both cases. The suspicion of there being a short umbilical cord cannot be used to explain the related pathological situation because the variation of the resistance to flow is monotonic with length changes, and thus measurements in umbilical arteries with short lengths yield low values of DI.

Figure 6 shows that blood flow is nearly independent of wall elasticity whereas DI may change within a discernible range. The wall properties of arteries as well as their thicknesses change with age, and their elasticity usually decreases with time. However, the umbilical arteries are free of degenerative vascular diseases (25), and thus it is reasonable to assume that any such changes in their wall properties and the corresponding expected DI will be insignificant.

Blood properties. Variation in blood viscosity induces changes in the expected DI similar to those induced by changing arterial length because both have a similar contribution to the resistance to blood flow (the denominator of Poiseuille’s law for laminar flow). Increasing blood viscosity increases the DI (Fig. 7); however, extreme changes in blood viscosity occur very rarely. For example, in abnormal conditions such as marked anemia or Rh isoimmunization, blood viscosity may decrease and, as a result, smaller values of DI are to be expected. On the other hand, the density of blood has very little effect on blood flow rate but induces changes in the expected DI (Fig. 7) like those for different diameters of the artery (Fig. 5) because it appears in the expressions for the pulsatile component of blood velocity (Eqs. 8 and 9).

Blood pressure. The influence of blood pressure was examined from different aspects because the umbilical arterial pressure was assumed to be composed of an oscillating component superimposed on a constant pressure level that decreases along the artery toward the placenta. Examination of the effect of maximal blood pressure at the fetal end of the umbilical artery (umbilicus) was conducted by variation of the fundamental harmonic of the unsteady component with negligible changes in mean blood pressure and blood flow rate (Fig. 8). Figure 8 also shows that FHR does not affect...
blood flow rate and the DI. A similar result was also obtained in other models that used a sinusoidal function for the pressure wave rather than a realistic signal (26). Nevertheless, DI measured from the umbilical artery tend to decrease slightly with FHR over the normal clinical range (26).

A different perspective was studied by changing the level of mean blood pressure at the ends of the umbilical artery. First, we changed the mean pressure either at the umbilicus \( P_{0,u} \) or at the placenta \( P_{0,p} \), which yielded considerable variations in the blood flow rate and the DI (Fig. 9) because of the steady pressure drop along the artery. We then changed the mean pressure simultaneously in the umbilicus and the placenta sites of the umbilical artery in such a way that \( P_{0,p}/P_{0,u} = 0.5 \) was held constant. These changes also induced alterations in blood flow rates and the DI, but within a much smaller range (Fig. 9) because the absolute values of blood pressure gradient are smaller than in cases where \( P_{0,p} \) is held fixed with increasing values of \( P_{0,u} \).

Abnormal waveforms. In clinical practice, the shape of the FVW is also examined for cases with the absence of end-diastolic flow or the presence of a postsystolic notch. In this study, we simulated irregular FVW by changing the spectral content of the input pressure wave. In particular, we were interested in simulations of a postsystolic notch on the FVW, which usually reflects a high distal resistance to umbilical blood flow even in cases with normal DI. For this purpose, we conducted simulations of cases in which we changed the magnitude of some harmonics (either separately or in combinations) and found that increasing the ratio between the second harmonic and the first harmonic generates a postsystolic notch on the FVW. The values of DI from the FVW with the notch were similar to normal values except for S/D. Moreover, we conducted simulations with this abnormal FVW and varied the diameter of the umbilical artery (as shown for normal FVW in Fig. 5) and obtained the same variability of the DI as with normal FVW.

In conclusion, a hemodynamic model of pulsatile blood flow in the umbilical artery was used to study the contribution of anatomic and physiological parameters on the variability of DI and their correlation with fetal blood flow rate. The simulated FVW allowed analysis of the sensitivity of DI to the site of Doppler measurements, which revealed that the DI is insensitive to the radial adjustment of the ultrasound beam as long as it is far away from the walls. However, the axial location is significant to the computed values of the DI, which supports the recommendations to measure DI at defined points (e.g., placental insert), a technique that allows reproducibility and comparison among data. This study also provided an insight into the dependence of DI on the anatomic and physiological characteristics of umbilical blood flow. The results showed that the values of DI vary linearly with arterial length, mean arterial pressure at the placental insert, blood viscosity, and maximal pressure in the umbilicus, whereas they vary inversely with arterial diameter, arterial thickness, module of elasticity, blood density, and mean arterial pressure in the umbilicus. The Doppler indexes as well as fetal blood flow rate are practically independent of FHR. Additional investigation is required to explore the cases that show changes in the values of DI while umbilical blood flow remains unaffected.

**APPENDIX A**

**Calculation of Doppler Indexes**

The Doppler indexes are dimensionless indexes defined by the extreme and mean velocities \( (W_{\text{min}}, W_{\text{max}}, W_{\text{mean}}) \) of the FVW. The Pourcelot's resistance index \((RI)\), the systolic-to-diastolic ratio \((S/D)\), and the pulsatility index \((PI)\) are given by (16)

\[
RI = \frac{W_{\text{max}} - W_{\text{min}}}{W_{\text{max}}} \quad (A1)
\]

\[
SD = \frac{W_{\text{max}}}{W_{\text{min}}} \quad (A2)
\]

\[
PI = \frac{W_{\text{max}} - W_{\text{min}}}{W_{\text{mean}}} \quad (A3)
\]

**APPENDIX B**

**Wave Propagation in a Fluid-Filled Elastic Tube**

Equilibrium equations for fluid. The conservation of continuity and momentum for axisymmetric, incompressible viscous fluid flow without body forces are given in polar coordinates \((r, \theta)\) by

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} = -\frac{1}{r} \frac{\partial p}{\partial r} + \frac{\partial \left(\frac{u}{r}\right)}{\partial \theta} + \frac{\partial \left(\frac{w}{r}\right)}{\partial \theta} - \frac{u}{r} \frac{\partial^2 u}{\partial z^2} - \frac{w}{r} \frac{\partial^2 u}{\partial z^2} \quad (B1)
\]

\[
\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} = -\frac{1}{r} \frac{\partial p}{\partial r} + \frac{\partial \left(\frac{w}{r}\right)}{\partial \theta} + \frac{\partial \left(\frac{u}{r}\right)}{\partial \theta} - \frac{u}{r} \frac{\partial^2 w}{\partial z^2} - \frac{w}{r} \frac{\partial^2 w}{\partial z^2} \quad (B2)
\]

where \(p_f\) and \(v\) are the fluid density and kinematic viscosity, respectively.

Equations of motion of tube. The tube wall is thin compared with its diameter and is assumed to behave like a thin membranous shell. Accordingly, bending moments and shear stresses are negligible, and tensile stresses are uniformly distributed across its thickness. Because the problem is axisymmetric, any element of the tube is loaded only in the principal directions \((n, t)\) and undergoes displacements \(\xi\) and \(\eta\) in \(n\) and \(t\) directions, respectively (Fig. 2). The longitudinal \((t)\) and circumferential \((n)\) equilibrium equations are (4)

\[
\frac{\partial R}{\partial z} + \frac{\partial}{\partial z} \left(\frac{R_t}{R} T_t\right) = R \left(\frac{\partial R}{\partial z}\right)^2 - \frac{F_{\text{ext}}}{R_{\text{mean}}} = 0 \quad (B4)
\]

\[
\frac{\partial}{\partial z} \left(\frac{R_n}{R} T_n\right) = \frac{\partial^2 R}{\partial z^2} + \frac{\partial}{\partial z} \left(\frac{R_n}{R} T_n\right) = F_{\text{ext}} = 0 \quad (B5)
\]

where \(R\) is the tube radius. The external loads \(F_{\text{ext}}\) and \(F_{\text{ext}}\) are composed of inertia forces and the liquid forces at the
fluid-wall interface and are given by
\[ F_{\text{ext}} = -p_{1} h \left( \frac{\partial^{2} \xi}{\partial t^{2}} + \frac{\partial^{2} \eta}{\partial t^{2}} \right) \left( 1 + \left( \frac{\partial R}{\partial z} \right)^{2} \right) + \frac{1}{1 + \left( \frac{\partial R}{\partial z} \right)^{2}} \left( \frac{\partial R}{\partial t} \right)^{2} \right] \]
where \( F_{\text{ext}} \) is the force acting on the wall. \( F_{rr}, F_{zz}, \) and \( F_{rz} \) are components of the stress tensor for Newtonian fluid.

Tube constitutive law. The stress components \( T_{\xi} \) and \( T_{\eta} \) are related to the displacement components \( \xi \) and \( \eta \) by the expressions (4)
\[ T_{\xi} - T_{\xi_{0}} = \frac{Eh}{1 - \sigma^{2}} \left( \frac{\partial \xi}{\partial t} + \sigma \frac{\partial \eta}{\partial z} \right) \]
\[ T_{\eta} - T_{\eta_{0}} = \frac{Eh}{1 - \sigma^{2}} \left( \frac{\partial \eta}{\partial t} + \sigma \frac{\partial \xi}{\partial z} \right) \]

Boundary conditions. At the interface between fluid and the tube inner surface, the fluid particles and the wall are always in contact. Thus
\[ u(r, z, t)|_{r=R} = \frac{\partial \eta}{\partial t} \]
\[ w(r, z, t)|_{r=R} = \frac{\partial \xi}{\partial t} \]

The component of the fluid velocity perpendicular to the wall must be equal to the normal velocity of the inner surface of the tube. Because \( r = R(z,t) \) is the inner surface of the tube, this condition can be written as
\[ \frac{d}{dt} [r - R(z,t)] = 0 \]

Differential of Eq. B13a gives
\[ u - \frac{\partial R}{\partial t} + w \frac{\partial R}{\partial z} = 0 \]

Solution of governing equations. The problem wave propagation in the fluid-filled tube is governed by Eqs. B1–B12, which constitute a set of nonlinear equations. In cases in which the wavelength is much larger than the tube radius, the set of equations can be linearized without losing any significant component in the results. Accordingly, the dependent parameters are expanded into power series of the following form
\[ u = u_{1} e + u_{2} e^{2}, \ldots \]

In addition, it is assumed that \( u_{1}, w_{1}, p_{1}, \eta_{1} \) and \( \xi_{1} \) are harmonic waves, and thus
\[ f_{1} = f(r) \exp(i \omega(t - z/c)) \]

where \( f(r) \) is the amplitude and independent of \( r \) for \( \eta_{1} \) and \( \xi_{1} \).

Mathematical manipulation of the set of linearized equations yields the first-order approximation for \( u_{1}, \eta_{1}, \) and \( p_{1} \) (Eqs. 8–10).

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