A hemodynamic analysis of coronary capillary blood flow based on anatomic and distensibility data

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Kassab, Ghassan S., Kha N. Le, and Yuan-Cheng B. Fung. A hemodynamic analysis of coronary capillary blood flow based on anatomic and distensibility data. Am. J. Physiol. 277 (Heart Circ. Physiol. 46): H2158–H2166, 1999.—An understanding of cardiac health and disease requires knowledge of the various factors that control coronary capillary blood flow. An analysis of coronary capillary blood flow based on a complete set of actual data on the capillary anatomy and elasticity does not exist. Previously, a complete set of data on the branching pattern and the vascular geometry of the pig coronary capillary network were obtained in our laboratory. In the present study, we obtained distensibility data on the coronary capillary blood vessels on the epicardial surface in the form of a pressure-diameter relationship using intravital microscopy. A mathematical model of the coronary capillary blood flow was then constructed on the basis of measured anatomic and elasticity data of the coronary capillary network, rheology of blood, physical laws governing blood flow, and appropriate boundary conditions. The constructed model was used to examine the heterogeneity of the spatial distribution of coronary blood flow, which is an important issue in coronary physiology. One interesting result of the model is that the dispersions of pressure and flow are significantly reduced in the presence of capillary cross-connections, and the resistance to flow is reduced as well. Finally, we found that the compliance of the epicardial surface capillary vessels is so small that its effect on the blood pressure drop is negligible in the diastolic state. However, the compliance of the intramyocardial capillaries remains unknown, and the interaction of the muscle contraction and blood vessel elasticity in systole remains to be studied.

capillary cross-connections; pressure distribution; flow distributions; heterogeneity; compliance

THE SPATIAL DISTRIBUTION of blood flow into the coronary capillaries has obvious physiological significance because the nutrition of the heart muscle depends on the blood flow in the capillaries. Our hypothesis is that the branching pattern and vascular geometry of the coronary capillary network are important determinants of coronary capillary blood flow that, in turn, influence the transport of oxygen and nutrients to the myocardium. We have previously stressed that the topological structure of the coronary arteries and intramyocardial veins are treelike but that the coronary capillary blood vessels have a non-treelike topology (9–11). The capillaries not only branch but also cross-connect along their lengths (9). The presence of cross-connections in the myocardial capillaries may make the pressure and flow distributions in the capillary bed more uniform.

A network simulation of the coronary capillary blood flow has been previously done by Wieringa et al. (25) in a model of hexagonally stacked parallel capillaries with randomly distributed interconnections based on the experimental data of Bassingthwaighte et al. (2). Wieringa et al. (25) assumed that all capillaries have uniform diameter and obey a linear pressure-flow relationship that did not take into account the capillary distensibility and the non-Newtonian blood rheology.

We wanted to improve the existing analysis of the coronary capillary blood flow in four ways: 1) use the topology and vascular dimensions of the coronary capillary network as determined by morphometry; 2) measure the compliance or distensibility of the coronary capillaries and use the results in the network analysis; 3) incorporate the non-Newtonian blood rheology in the analysis; and 4) embed the capillary networks between coronary arterial and venous trees realistically according to morphometric data. With regard to 1 and 4 above, we refer to our previously measured morphometric data (9). With regard to 3 above, we refer to the work of Lingard (13), Lipowsky et al. (14), and Pries et al. (15). For 2 above, a new morphometric study on the distensibility of epicardial capillaries of the pig left ventricle at the diastolic state is described here. With these data we can use the laws of physics (conservation of mass and momentum) and the appropriate boundary conditions to formulate well-posed boundary value problems for the hemodynamics of the coronary capillary network. Extension to systolic state and the beating heart awaits future work. In the context of the present analysis, our main goals are to determine the effect of capillary cross-connections and the elasticity of vessels on the pressure and flow distributions in the coronary capillary network.

METHODS

Isolated heart preparation. The studies were performed on healthy farm pigs weighing 28–32 kg. Surgical anesthesia was induced with ketamine (33 mg/kg im) and atropine (0.05 mg/kg im) and maintained with pentobarbital sodium (30 mg/kg iv, in an ear vein). A midline sternotomy was performed, ventilation with room air was provided with a respiratory pump, and anticoagulation was induced with heparin (100 U/kg). An incision was made in the pericardium, and the heart was supported in a pericardial cradle. The heart was arrested with a saturated KCl solution given through a jugular vein. The heart was then excised, with the ascending aorta clamped to keep air bubbles out of the coronary vessels, and placed in a cold (0°C) saline bath. The right coronary artery, left anterior descending coronary artery (LAD), left circumflex artery (LCX), and coronary sinus...
artery (CS) were cannulated with saline to avoid air bubbles. These coronary arteries were then immediately perfused with an isosmotic, cardioplegic rinsing solution as described in Kassab et al. (11) to maintain the myocardium relaxed and the vasculature vasodilated.

Pressure-diameter relationship. To examine the distensibility of the epicardial surface capillaries, the surface of the isolated heart preparation was transilluminated and viewed with an intravital microscope. The coronary arteries were perfused with a colored microfil (inert, fluid silicone) to visualize the epicardial surface microvessels. The silicone microfil is a water-immiscible, long-chain polymeric material that does not extravasate the vessel. Also, no catalyst was added to the microfil polymer, so it remained a viscous fluid throughout the experiment. Once the microfil was observed in the CS and in the heart chambers, fast-hardening, catalyzed microfil (curing time 3–5 min) was poured into the heart chambers. The cast chambers seal off any arterial luminal or Thebesian drainage. Hence, the pressure was regulated in the entire vasculature by clamping off the CS and establishing a static pressure throughout the vasculature. Epicardial surface capillaries were identified and their elastic pressure-diameter relationship determined. Initially, the vessels were preconditioned with loading and unloading pressure ramps over several cycles. Subsequently, images were recorded in the pressure range from 0 to 60 mmHg in increments of 10 mmHg for the capillaries. Typically, images of the coronary epicardial surface vessels perfused with microfil are shown in Kassab et al. (10). Diameter measurements were made using a × 40 objective (NA = 0.77) with a resolution of 0.5 μm. An image grabber was used to grab the frames of a given capillary vessel at different pressures. The images were stored on a video floppy disk and analyzed at a later time. The grabbed frames were input into our image processing system, where the diameters of the vessels were digitized. The mean diameter, for each pressure, was computed from several diameter measurements made over a 20- to 30-μm length of the capillary vessel.

RESULTS

Compliance of capillary vessels. Figure 1A shows the pressure-diameter relationship of the 12 capillary vessels measured at the epicardial surface from 3 pig hearts (4 vessels from each heart). The measurements were made at the base of the heart near the LAD-LCX artery bifurcation. Figure 1B shows that, in the pressure range (10–50 mmHg), the elastic deformation can be described by the equation

\[ D - D^* = \alpha (P - P^*) \]  \hspace{1cm} (1)

where D is the diameter at a given intravascular pressure P, D* is the diameter corresponding to the physiological pressure P* (30 mmHg), and \( \alpha \) is the compliance constant of the vessel. The mean ± SD of \( \alpha \) over the 12 measurements was found to be 1.7 ± 0.91 × 10^{-6} cm/mmHg by a linear least-squares fit of the data with the intercept set at zero, as shown in Fig. 1B, in the 10–50 mmHg pressure range with a mean correlation coefficient of 0.92.

Simulation analysis of coronary capillary blood flow based on morphometric, rheological, and compliance data. Kassab and Fung (9) designated all capillaries as blood vessels of order number zero; we further designated the capillaries as those fed directly by arterioles (\( C_{0a} \)), those drained directly into venules (\( C_{0v} \)), and those connected to \( C_a \) and \( C_v \) (\( C_0 \)). The capillaries branch in patterns identified as Y, T, H, or HP (hairpin) on the basis of their geometric shape and anastomose through capillary cross-connections (\( C_{cc} \)). The \( C_{cc} \) vessels may connect adjacent capillaries or capillaries originating from different arterioles. A branching pattern of the coronary capillary bed was constructed on the basis of these patterns (the frequency of Y, T, H, and HP patterns simulated the measurements in Ref. 9), whereas the vascular dimensions (diameters and lengths) were prescribed by the morphometric data of Kassab and Fung (9). An idealized case is shown in Fig. 2.

The vascular geometry and the flow condition justify the assumptions that the Reynolds and Womersley numbers of the flow are very small (<<1) and that the length-to-diameter ratio of each capillary vessel is large. Under these assumptions, the classic Poiseuille’s law can be used to describe the local pressure-flow relationship in a circular cylindrical capillary tube as

\[ \frac{dP}{dx} = -\left( \frac{128\mu L}{\pi D^4} \right) Q \]  \hspace{1cm} (2)

where Q is the volume-flow rate, x is the axial coordi-
nate measured from the entrance section of each capillary, and D and \( \mu \) are the local diameter and coefficient of viscosity, respectively (7).

At the capillary dimension, the particulate nature of the blood cells becomes important and the blood properties become non-Newtonian. The viscosity in Poiseuille’s law (Eq. 2) is no longer constant and should be considered as an apparent viscosity (\( \mu_{\text{app}} \)). In general, the apparent viscosity is a function of vessel diameter, hematocrit, and shear strain rate. The dependence of apparent viscosity on vessel diameter and shear rate was given by Pries et al. (15). They proposed a modified viscosity law based on a compilation of literature data on relative blood viscosity in in vitro and in vivo tube flow experimental measurements. In our range of capillary diameters (2.5–9.0 \( \mu \text{m} \)), the apparent viscosity \( \mu_{\text{app}} \) is a function of vessel diameter, shear rate, and the shear rate. We chose \( k_1 = 1.81 \text{ dP/s}^{10} \) and \( k_2 = 1.04 \text{ (dP/s)}^{10} \), corresponding to a tube diameter of 5.34 \( \mu \text{m} \) and a tube hematocrit of 39% (13). Hence, we computed the apparent viscosity using Eq. 3b to obtain an updated apparent viscosity. This process was iterated until the viscosity converged for those vessels with a shear rate <50 s\(^{-1}\).

The hydrodynamic law (Eq. 2) can be combined with the elasticity (Eq. 1) and rheology of blood (Eq. 3) relationships to yield

\[
dP/dx = 128/\pi a_0 + b_2 [\alpha(P - P*) + D*] + c_2 [\alpha(P - P*) + D*]^2 + d_3 [\alpha(P - P*) + D*]^4 Q
\]

for vessels with a shear rate >50 s\(^{-1}\) and

\[
dP/dx = 128/\pi [(k_1 + k_2)/\pi \alpha(P - P*) + D*]^2 [\alpha(P - P*) + D*]^4 Q
\]

for vessels with a shear rate <50 s\(^{-1}\). These are the governing equations for non-Newtonian viscous blood flow in an elastic vessel at steady-state conditions that can be integrated for specific boundary conditions.

Effect of \( C_{cc} \) on the hemodynamics of the capillary network. To examine the effect of \( C_{cc} \) on the pressure and flow distributions in the capillary bed, we assumed that the compliance was zero (\( \alpha = 0 \)) and considered a special case of Eq. 4. The problem was formulated as follows. In a network of capillaries (see example in Fig. 2), the nodes were numbered as 1, 2, ..., M. In a vessel connecting two nodes represented as i and j, the flow from node i to node j was denoted as \( Q_{ij} \), whereas the differential of pressures at nodes i and j was denoted as \( \Delta P_{ij} \). The pressure-flow relationship was then written as

\[
Q_{ij} = (\pi/128) \Delta P_{ij} G_{ij}
\]

where

\[
\Delta P_{ij} = P_i - P_j
\]

and

\[
G_{ij} = (D_{ij})^{4/3} / (L_{ij} \mu_{\text{app}ij})
\]

is a quantity called the vascular conductance of the vessel ij, which is a function of \( D_{ij} \), \( L_{ij} \), and \( U_{ij} \), the diameter, length, and velocity, respectively, between nodes i and j. The expression for \( \mu_{\text{app}ij} \) is given by Eq. 3.

Figure 2 shows that there are three vessels emanating from the jth node anywhere in the network. By denoting these three vessels as ij with i = 1, 2, and 3, and by conservation of mass, we obtained

\[
\sum_{i=1}^{3} Q_{ij} = 0
\]

in which the volumetric flow into a node is considered positive and that out of a node is negative. From Eqs. 5 and 6 we obtained a set of nonlinear algebraic equations in pressure for M nodes in the network (\( M = 136 \)) and pressure for the cores of the vessels (\( M = 136 \)).
nodes for the simulated network in Fig. 2), namely

\[ \sum_{i=1}^{3} (P_i - P_j)G_{ij} = 0 \]  

(7)

The set of equations represented by Eq. 7 can be reduced to a set of simultaneous algebraic equations that are solved iteratively for the nodal pressures once the conductances are evaluated from the geometry and suitable boundary conditions are specified. The boundary conditions are simulated by a random uniform distribution with the range

\[
\begin{align*}
P(\text{at inlet of } C_{0a}) &= 32-42 \text{ mm Hg} \\
P(\text{at outlet of } C_{0v}) &= 26-28 \text{ mmHg}
\end{align*}
\]  

(8a)

These boundary conditions represent a mean pressure drop of 10 mmHg, which is in agreement with the micropressure measurements of Klassen et al. (12) on the epicardial surface of the dog left ventricle. The arterioles and venules were chosen randomly, always maintaining a ratio of four arterioles to seven venules, consistent with our previous morphometric measurements (10, 11). In matrix form, this set of equations is

\[ GP = G'P' \]  

(8b)

where \( G \) is the \( M \times M \) matrix of conductances, \( P \) is a \( 1 \times M \) column vector of the unknown nodal pressures, and \( G'P' \) is the column vector of the boundary pressures times the conductances of their attached vessels. Equations 8a and 8b are solved by the Gaussian elimination method. The capillaries indicated by asterisks in Fig. 2 connect to capillary vessels above and below the capillary plane. We initially assigned these vessels an arbitrary pressure value and then updated their values according to the mean values of pressures in nearby capillaries. We found that these pressures converge within several runs of the model.

Because we have the raw data for the diameters and lengths of the four orders of capillaries (\( C_{0a} \), \( C_{00} \), \( C_{0v} \), and \( C_{cc} \)), we input them directly to the computation to avoid an intermediate step of constructing a random number table (satisfying the means and standard deviations of Table 2 in Ref. 9). We numbered the entries in our data file, for each order, in positions 1, 2, 3, ..., \( N \), where \( N \) is the total number of measurements for that order. If the total number of elements in the computational model exceeded \( N \), then the program used the data sets repeatedly in sequence until the required number of elements was obtained. This algorithm is simple and as accurate as that using the raw data. To account for the randomness by which the measured data are arranged in sequence, and to use the whole distributions, we ran the program repeatedly and varied the starting point in sequence for each order.

The solution satisfying Eq. 8 is obtained in the form of a column vector of the nodal pressures throughout the arterial network. The pressure drops as well as the corresponding flows were computed. Figure 3, A–D, shows the log-transformed, median-normalized flow distribution in capillaries of orders \( C_{0a} \), \( C_{00} \), \( C_{0v} \), and \( C_{cc} \), respectively. These distributions correspond to data obtained over 100 runs of the model. We examined the effect of varying the number of runs of the model and found that the median flows obtained from 100 runs were within 1.5% of the median flows obtained from 1,000 runs. Hence, we opted to run the numerical program 100 times to obtain all the results shown here. Figures 4A and 5A show the effect of \( C_{cc} \) on the median flows and pressure drops per capillary segment, respectively, for all orders of capillaries. Figures 4B and 5B show the effect of \( C_{cc} \) on the coefficient of variance (CV; SD/mean, %) of the flows and pressure drops, respectively. The effect of \( C_{cc} \) on the total flow into the capillary network (\( QT_{in} \)) is shown in Fig. 6. \( QT_{in} \) is normalized with respect to the total flow into the network in the presence of all \( C_{cc} \). Finally, the effect of cross-connections on the median velocity of various capillary orders is shown in Fig. 7.

Effect of vessel compliance on the hemodynamics of the capillary network. With the distensibility of the blood vessels known, the mechanics of the blood vessel were coupled to the mechanics of blood flow to yield a pressure-flow relationship for each vessel segment. This can be demonstrated as follows. In a stationary, nonpermeable tube, \( Q \) is a constant throughout the length of the tube, whereas the tube diameter and the apparent coefficient of viscosity are functions of \( x \) because of the elastic deformation. The elastic deformation of the coronary capillaries can be described by Eq. 1, which can be differentiated to yield

\[ \frac{dP}{dx} = \frac{dP}{dD} \frac{dD}{dx} = \left(1/\alpha \right) \frac{dD}{dx} \]  

(9)

By substituting Eq. 9 into Eq. 2 and assuming \( \mu_{app} \) to be a constant for each individual order (5.7 cP for the order \( C_{0a} \), 5.0 cP for \( C_{00} \), 5.5 cP for \( C_{0v} \), and 5.1 cP for \( C_{cc} \) as the computed means from our model), we obtained

\[ D^4 \frac{dD}{dx} = -\left(128\mu_{app} \alpha Q/\pi \right) \]  

(10)

Because the right-hand term is a constant independent of \( x \), we obtained the integrated result

\[ D^5(x) = -\left(640\mu_{app} \alpha Q/\pi \right) x + D^5(0) \]  

(11)

The integration constant is determined by the boundary condition stating that when \( x = 0 \), \( D(x) = D(0) \). Solving Eq. 2 for \( P(x) \) with \( D = D(x) \) given by Eq. 11 yielded a nonlinear pressure-flow relationship for each capillary segment that takes the form (see APPENDIX)

\[ \Delta P_n = \left[ -D_n + \left( D_n^2 + 8\alpha_n \Delta P_{p} / D_n \right)^{1/2} \right] / 4\alpha_n \]  

(12)

where \( \Delta P_p \) is Poiseuille's pressure drop as given by Eq. 2 and \( n \) is the order number of the capillary. Equation 12 can be used instead of Poiseuille's law (Eq. 2) in the various segments to synthesize the pressure and flow distributions in the capillary network.
DISCUSSION

Effect of C_{cc} on the hemodynamics of the capillary network. One of the main goals of the present study was to examine the effect of C_{cc} on the pressure and flow distributions in the capillary network. We investigated this aspect by constructing a typical capillary network based on measured branching pattern and vascular geometry (diameters and lengths) and applied the laws of fluid dynamics to predict the flow behavior in the network.

Fig. 3. Log-transformed, median-normalized flow distributions in capillaries fed directly by arterioles (C_{0a}; A), capillaries connected to C_{0a} and C_{0v} (C_{00}; B), capillaries drained directly into venules (C_{0v}; C) and capillary cross-connections (C_{cc}; D). Q_{0a}, Q_{00}, Q_{0v}, and Q_{cc}, flow in C_{0a}, C_{00}, C_{0v}, and C_{cc}, respectively; Q_{median}, median flow.

Fig. 4. A: relationship between median flow per capillary segment and capillary order number, with and without C_{cc}. B: relationship between coefficient of variance (CV; SD/mean, %) of blood flow and capillary order number, with and without C_{cc}.
of physics and the appropriate boundary conditions to analyze the pressure and flow distributions with and without Ccc. Ccc were removed from the network by setting their conductances equal to zero. Our results show that the median flow and pressure drop increased in the presence of Ccc, as shown in Figs. 4A and 5A. The total flow into the capillary network also increased in the presence of Ccc, as shown in Fig. 6. The increase in total flow occurred for the same pressure drop across the capillary network. Hence, the resistance to flow was decreased by the presence of Ccc. Furthermore, Ccc also homogenized the flow and pressure distributions. Figures 4B and 5B show that the relative dispersion (or CV) of flow and pressure drop was reduced in the presence of Ccc. Although these dispersions were reduced in the presence of Ccc, the dispersions at the venous capillaries were still greater than those at the arterial capillaries, that is, the capillary outlet flow and pressure are more heterogeneous than the inlet flow and pressure with or without Ccc. This result may stem from the fact that the number of venous capillaries is greater than the number of arterial capillaries (10). Hence, an increase in the number of possible pathways may lead to an increase in the variability of the hemodynamic parameters.

The median blood velocity shown in Fig. 7 was computed from the flows and cross-sectional areas of the capillary vessels. The median velocities range from ~300 µm/s in capillaries of order C00 to 1,400 µm/s in capillaries of order C0a. Tillmanns et al. (24) found average diastolic capillary red cell velocities of 909 and 1,428 µm/s on the epicardial surface of turtle and dog left ventricles, respectively. Direct comparison cannot be made, however, because the order of capillary vessels is unspecified in the measurements of Tillmanns et al. Wieringa et al. (25) found a mean value of 1,428 µm/s in their capillary network simulation.

Once the hemodynamics of the network have been determined, Poiseuille’s hypothesis can be reexamined. Poiseuille’s law applies only when the flow has low Reynolds and Womersley numbers. The former is defined by the formula $U D / n$, where $U$ is the mean velocity of flow, $D$ is the blood vessel lumen diameter, and $n$ is the kinematic viscosity of blood. The latter is defined as $(D/2)(w/ν)^{1/2}$, where $ω$ (in radians/s) is the circular frequency of pulsatile flow and is computed for a heart.
rate of 100 cycles/min. We have found that the median Reynolds and Womersley numbers are 0.002 and 0.006, respectively, for all orders of capillary vessels and hence justify the steady-state assumption.

Effect of vessel compliance on the hemodynamics of the capillary network. The pressure drop in Eq. 12 can be plotted as a function of the compliance constant for the various orders of capillary vessels as shown in Fig. 8. When the compliance is zero (rigid vessel), the pressure drop corresponds to that given by Poiseuille's equation. However, when the compliance is nonzero, the pressure drop is smaller than that given by Poiseuille's equation and varies for the different orders of capillary vessels. For our measured values of compliance ($7.2 \times 10^{-7} - 3.5 \times 10^{-6}$ cm/mmHg), the pressure drop predicted by Eq. 12 is nearly equal to Poiseuille's pressure drop as shown in Fig. 8. Hence, we concluded that the effect of epicardial surface capillary compliance on the hemodynamics of the capillary network is negligible in the physiological range of pressures. However, these are only the epicardial surface vessels, and it is unknown whether the intramyocardial vessels are equally stiff.

Comparison with other capillary distensibility data. The distensibility of capillary blood vessels was previously determined using several methods: airtight pressure chamber for the bat wing (4), microannulation and injection of oil drops (23), microocclusion within the limits of a pulse pressure range (18), and elastomer perfusion under known hydrostatic pressures (20). Table 1 summarizes the data for the distensibility of the capillary vessels in various organs and species. It can be seen that the epicardial coronary capillaries are among the least distensible vessels in various organs. Unlike coronary and systemic capillaries, pulmonary capillaries are very distensible because they receive little support from the surrounding tissues (6).

Model limitations. Although our analysis is based on measured morphometric and elasticity data for coronary capillary blood vessels, there were still a number of assumptions made. For example, the distensibility data were obtained from the epicardial surface only because intramural vessels could not be readily visualized with the present technique. Furthermore, the topology of the capillary branching at the epicardial surface is different from that of intramural layers, where the morphometric data were measured. Hence, we combined morphometric data from intramural capillaries with distensibility data from epicardial surface capillaries. Furthermore, we did not measure the distensibility of various orders of capillaries. The findings in the bat wing suggest the existence of a longitudinal gradient of distensibility in the capillary compartment (3). We also restricted our hemodynamic analysis to the diastolic state of the myocardium. Hence, we did not consider the muscle-vessel interaction that is very important in systole. Muscle tension and contraction may have a large effect on capillary diameters and may change the capillary transmural pressure so that the

![Fig. 8. Relationship between pressure drop per vessel segment ($\Delta P$) and logarithm of compliance constant for various orders of capillary vessels. Pressure drop is normalized with respect to Poiseuille's pressure drop ($\Delta P_P$).](image)

Table 1. Capillary distensibility in various organs and species

<table>
<thead>
<tr>
<th>Species</th>
<th>Organ</th>
<th>Distensibility, $(\text{mmHg}) \times 10^{-3}$</th>
<th>Pressure, mmHg</th>
<th>Method</th>
<th>Reference</th>
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<tbody>
<tr>
<td>Dog</td>
<td>Lung</td>
<td>64.9</td>
<td>0–18.8 (7.5)</td>
<td>Freezing technique</td>
<td>8</td>
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<tr>
<td></td>
<td></td>
<td>42.0</td>
<td>0–18.8 (18.8)</td>
<td></td>
<td></td>
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<tr>
<td>Cat</td>
<td>Lung</td>
<td>68.4</td>
<td>4.7–20.6 (7.5)</td>
<td>Elastomer technique</td>
<td>19</td>
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<tr>
<td>Human</td>
<td>Lung</td>
<td>48.3</td>
<td>&lt;24.1</td>
<td>Elastomer technique</td>
<td>21</td>
</tr>
<tr>
<td>Cat</td>
<td>Mesentery</td>
<td>1.20</td>
<td>Pulse</td>
<td>In vivo cinemicrography</td>
<td>18</td>
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<td>1.90</td>
<td>0–50</td>
<td>Elastomer technique</td>
<td>17</td>
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<tr>
<td>Bat</td>
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<td>7.5–9.4</td>
<td>48–60</td>
<td>Box technique</td>
<td>4</td>
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<tr>
<td>Frog</td>
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<td>1.89</td>
<td>10</td>
<td>Oil drop technique</td>
<td>1</td>
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<td></td>
<td></td>
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<td>0.76</td>
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<td>Frog</td>
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<td>10</td>
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<td>22, 23</td>
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<td></td>
<td></td>
<td>1.84</td>
<td>30</td>
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<td>Frog</td>
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<td>Oil drop technique</td>
<td>22, 23</td>
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<td>Wing</td>
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<td>50</td>
<td>Box technique</td>
<td>3</td>
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<td></td>
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<td>4.5</td>
<td>100</td>
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<td>10–50</td>
<td>Elastomer technique</td>
<td>Present study</td>
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Distensibility is defined as $(\Delta D / D) / \Delta P$, where $D$ and $P$ are diameter and pressure, respectively. Pressure values given in parentheses are transpulmonary pressure.
pressure-diameter relationship shown in Fig. 1 is no longer applicable. Furthermore, we did not consider the time-varying vasoactive components of the arterioles that would make the boundary conditions time dependent. These important effects of muscle-vessel interactions must be investigated in the future. The present approach is only a logical first step.

Fibich et al. (5) formulated a continuum mechanics model of the coronary capillary vessel throughout the cardiac cycle. They modeled the capillary network as a single long tube with \( P_r \) and \( P_w \) as the inlet arteriole and outlet venous pressures, respectively. Their model considers the effect of permeability, distensibility, and tissue stress on the basis of a number of hypotheses. Their results show that under physiological conditions ultrafiltration is of minor importance, and its effect was neglected in the present study. Their analysis also predicts regional differences in capillary flow. It would be interesting to combine their time- and space-dependent flow equation with our network model using measured capillary diameters, lengths, and compliance. This would yield a system of nonlinear partial differential equations for pressure whose solution would vary transmurally and throughout the cardiac cycle.

In conclusion, our network analysis has shown that the capillary cross-connections tend to homogenize the pressure and flow distributions and reduce the flow resistance of the capillary network. Hence, the analysis clarifies an important hemodynamic function of the capillary cross-connections, showing that the cross-connections play an important role in the structure-function relationship. Furthermore, our measured compliance of the epicardial surface coronary capillary vessels is relatively small in the physiological pressure range. Our analysis has shown that the effect of the measured epicardial surface vessel compliance on the hemodynamics of the coronary capillary is negligible in the diastolic state of the heart. The compliance of intramyocardial blood vessels remains unknown, however, and the interaction of the muscle contraction and blood vessel elasticity in systole remains to be studied.

**APPENDIX**

The mathematical steps between Eqs. 11 and 12 are given below. We let \( x = 0 \) be the entry section and \( x = L \) be the exit section of a capillary. Letting \( x = L \) in Eq. 11 yielded

\[
D^2(L) - D^2(0) = -640 \mu_{app} \alpha Q L / \pi \tag{13}
\]

We next sought an approximate expression of Eq. 13 for which \( D(L) - D(0) \) is small. By letting \( D(L) = D(0) + \epsilon \), expanding the left-hand side of Eq. 13 in a power series of \( \epsilon \), and retaining only terms up to \( \epsilon^2 \), we obtained the approximation

\[
[D(L) - D(0)]^2 + 2[D(L) - D(0)]D(0) = (-128 \mu_{app} \alpha L Q) / [\pi D^4(0)]
\]

Using Eq. 1 first at \( x = L \) and then at \( x = 0 \) and subtracting, we obtained

\[
D(L) - D(0) = a[P(L) - P(0)] \tag{15}
\]

By combining Eqs. 14 and 15 and writing \( D_0 \) for \( D(0) \), we obtained

\[
\Delta P + (2aD_0) \Delta P^2 = -128 \mu_{app} L Q / \pi D_0^4 \tag{16}
\]

where \( \Delta P = P(L) - P(0) \). The solution to Eq. 16 is Eq. 12.

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