Transmural gradients of cardiac myofiber shortening in aortic valve stenosis patients using MRI tagging

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Van der Toorn, A., P. Barenbrug, G. Snoep, F. H. Van der Veen, T. Delhaas, F. W. Prinzen, J. Maessen, and T. Arts. Transmural gradients of cardiac myofiber shortening in aortic valve stenosis patients using MRI tagging. Am J Physiol Heart Circ Physiol 283: H1609–H1615, 2002.—Aortic valve stenosis impairs subendocardial perfusion with a risk of irreversible subendocardial tissue damage. A likely precursor of damage is subendocardial contractile dysfunction, expressed by the parameter TransDif, which is defined as epicardial minus endocardial myofiber shortening, normalized to the mean value. With the use of magnetic resonance tagging in two short-axis slices of the left ventricle (LV), TransDif was derived from LV torsion and contraction during ejection. TransDif was determined in healthy volunteers (control, n = 9) and in patients with aortic valve stenosis before (AVSten, n = 9) and 3 mo after valve replacement (AVRepl, n = 7). In the control group, TransDif was 0.00 ± 0.14 (mean ± SD). In the AVSten group, TransDif increased to 0.96 ± 0.62, suggesting impairment of subendocardial myofiber shortening. In the AVRepl group, TransDif decreased to 0.37 ± 0.20 but was still elevated. In eight of nine AVSten patients, the TransDif value was elevated individually (P < 0.001), suggesting that the noninvasively determined parameter TransDif may provide important information in planning of treatment of aortic valve stenosis.

aortic valve replacement; function; torsion

MOST CARDIAC DISORDERS affect motion of the left ventricular (LV) wall (18, 19, 25). In aortic valve stenosis, coronary flow diminishes in the subendocardial layers relative to subepicardial ones (9, 16, 26). Ultimately, this may lead to subendocardial infarction and other structural changes (13–15). The risk to run into permanent damage is the most important reason for aortic valve replacement. Currently, this risk is estimated by combined assessment of hemodynamic parameters (peak systolic LV pressure, transvalvular pressure drop, and peak flow velocity), general LV pump function parameters (ejection fraction and maximum time derivative of LV pressure), effects of increased venous pressure (edema), and sound from turbulent flow. In the search for a clear indicative parameter, measurement of subendocardial contractile function may be promising. After all, underperfusion of myocardial tissue results in an immediate decrease of contractile function (10, 22) far before irreversible tissue damage occurs. The present study was conducted to investigate whether measurement of subendocardial contractile dysfunction is possible and whether it may provide additional clinically valuable information.

Commonly, myocardial contractile dysfunction is detected from wall motion abnormalities, as measured by echocardiography. In aortic valve stenosis patients, such abnormalities cannot be found easily, probably because the related contractile dysfunction is distributed smoothly over the whole inner side of the LV wall. In the last decade, the MRI-tagging technique has been developed to assess myocardial motion noninvasively (18, 20, 27). With this technique, during a period of a few milliseconds, the magnetization of the tissue is periodically modulated with a grid pattern, thus forming tags. During the cardiac cycle, the tags are moving with the tissue and are visualized with MRI. With the use of image analysis techniques, the tags are followed in time. Thus a map of motion of the visualized cross section is obtained. In the current study, we propose to use MRI tagging in detecting transmural differences in myocardial contractile function.

In model studies on cardiac mechanics (3, 5), it was predicted that a transmural gradient in contractile function was related to changes in the ratio of torsion to shortening (TSR) of the LV. Torsion is defined here as a base-to-apex gradient of rotation of the short-axis cross section around the LV long axis. When assuming myofiber shortening to be transmurally uniform, TSR was predicted to be a fixed number. In dogs with two-dimensional echocardiography (3) and in humans with MRI tagging (1), this prediction was confirmed. In humans using MRI tagging, the transmural relative variation of myofiber shortening was reported to be <10% (17, 23). In aortic valve stenosis patients, torsion found was to be elevated (21, 24). Torsion also changed...
by changes in preload, afterload, or contractility (8). We conclude that under normal conditions myofiber shortening is quite evenly distributed over the LV wall, a finding that was confirmed experimentally by finding a fixed ratio of TSR. During aortic valve stenosis, torsion may be modified by changes in the transmural distribution of myofiber shortening. In the present study, we investigated whether a parameter expressing a transmural difference in myofiber shortening can be derived from myocardial motion, as measured by MRI tagging.

In the normal heart, due to the specific helical pathways of the myofibers, torsion as induced by the subepicardial myofibers is partly counteracted by the subendocardial myofibers. With aortic valve stenosis, subendocardial myofibers are likely to be less active, causing torsion to increase. Because fewer fibers are fully active, ejection is expected to decrease. Thus TSR is expected to be sensitive to a decrease of myofiber shortening in the subendocardium relative to the subepicardium. If load of the heart changes by physiological mechanisms such as exercise or contractility variation, myofiber shortening is expected to change uniformly throughout the heart. Both torsion and shortening are then expected to change proportionally, leaving TSR unaffected.

In the present study, TSR was determined from the motion of MRI tags in two parallel short-axis cross sections. With the use of a model of LV mechanics, from the TSR value, the transmural difference in myofiber shortening (TransDif), as normalized to the mean value, was estimated. Measurements were performed in healthy volunteers (control) and in aortic valve stenosis patients before (AVSten) and 3 mo after aortic valve replacement (AVRepl). It was investigated whether TSR and TransDif were affected by aortic valve stenosis. Finally, we investigated whether subendocardial myofiber shortening recovered after aortic valve replacement.

METHODS

Patient study. The study was conducted on 9 healthy volunteers (2 women and 7 men, age 31 ± 4 yr) and 10 patients (2 women and 8 men) with severe aortic valve stenosis, characterized by an aortic valve pressure gradient >50 mmHg. The AVSten group of patients (age 64 ± 9 yr) contained nine patients before aortic valve replacement. The AVRepl group of patients (age 68 ± 6 yr) contained seven patients 3 mo after aortic valve replacement. From the latter group, six individuals also belonged to the AVSten group. Written informed consent was obtained from all subjects.

Image acquisition. MRI experiments were performed at either 0.5 or 1.5 T (Philips Gyroscan T5 II or Gyroscan NT, Philips Medical Systems; Best, The Netherlands). All images were acquired using ECG triggering while patients were breathing normally. Images of two short-axis cross sections of the heart were obtained, ~2 and 4 cm below the base. Sixteen regular cine images per slice were acquired using a nontagged steady-state gradient echo sequence, starting 20 ms after the ECG trigger on the peak of the R wave and continuing with time intervals of either 20 or 62 ms (echo time 10 ms, slice thickness 8 mm, field of view 213 mm, image size 256 × 256 pixels). Thereafter, a series of tagged images from the same two slices were obtained with time intervals of 20 ms, using spatial modulation of the magnetization (6). Initial intertag distance was 5 mm with a tag width of 2.5 mm in both horizontal and vertical directions. To obtain a sufficient contrast-to-noise ratio for a longer time interval, two overlapping series of tagged images were acquired for each slice. Acquisition of the first set of MRI-tagged images was started 20 ms after the ECG trigger. The second set of MRI-tagged images started 130–160 ms later.

Image analysis. MR images were analyzed to quantify torsion and the decline in cavity area, as normalized to wall area. Image analysis was performed off-line using homemade software for Matlab 5.3.1 (MathWorks; Natick, MA). For both slices, a nontagged reference image was chosen at −160–180 ms after the ECG trigger. In this image, the LV wall was manually outlined for both slices (Fig. 1). Papillary muscles were considered not to be part of the LV wall. MRI images were recorded in the period from 30 ms until ~260 ms after the ECG trigger with regular time intervals of 20 ms. Each image frame was split into an image with horizontal and an image with vertical tag lines by band filtering in the frequency domain around the first harmonic frequency of the tag pattern (spatial frequency 0.2 mm⁻¹, ratio of bandwidth to center frequency 1.0). With the use of a correlation method previously applied for pulsed ultrasonic echo signals (7), from the successive images with horizontal tag lines, vertical displacement was calculated in each pixel to obtain vertical displacement maps for each time interval. Similarly, the images with vertical tags were used to obtain horizontal displacement maps. Because the boundaries of the LV wall move with the tags, the calculated displacement maps were used to determine the moving contours of the LV wall during the cardiac cycle. Thus the cross-sectional areas of the cavity (A_cav) and of the LV wall (A_wall) were determined as a function of time for slice 1 and slice 2, which were located at the basal and apical side of the equator, respectively. Shortening was quantified as the change (Δ) in the logarithm of the normalized inner diameter (ε_inner). It holds that

\[
ε_{\text{inner}} = \frac{1}{2} \ln \left( \frac{A_{\text{slice 1}}}{A_{\text{slice 2}}} \right) \div \left( \frac{A_{\text{wall 1}}}{A_{\text{wall 2}}} \right)
\]

Time-dependent rigid body rotations of slice 1 and slice 2 were also derived from the displacement maps. Torsion was defined as the shear angle on the epicardial surface between both slices (3). The time course of torsion (T) was calculated.

![Fig. 1. MRI cine image (A) and MRI-tagged image (B) of a healthy volunteer 178 ms after the peak R wave of the ECG. The wall, as delineated manually in the cine image, was used for further motion analysis.](http://ajpheart.physiology.org/doi/abs/10.1152/ajpheart.01374.2001)
from the axial gradient in rotation angle $\alpha$ multiplied by the outer radius ($R_{epi}$) using the approximation

$$T = \frac{(\alpha_{\text{slice} 1} - \alpha_{\text{slice} 2})}{d} \times \frac{(R_{\text{epi}} 1 + R_{\text{epi}} 2)}{2}$$  \hspace{1cm} (2)

with

$$R_{\text{epi}} = \sqrt{\frac{A_{\text{wall}} + A_{\text{cav}}}{\pi}}$$

The parameter $d$ represents the distance between both slices. By plotting torsion as a function of $\varepsilon_{i\text{inner}}$, a practically linear relationship was found (1) during ejection. In determination of the ejection phase, the beginning ejection usually started $\sim$40 ms after the ECG trigger. The end of ejection occurred $\sim$220 ms later, as marked by the end of decline of cavity area. The TSR was calculated as the negative slope of the linear regression line of torsion as a function of $\varepsilon_{i\text{inner}}$ during the ejection phase

$$T = -\text{TSR} \times \varepsilon_{i\text{inner}} + \text{intercept} + \text{error} \hspace{1cm} (3)$$

The intercept has not been used for further analysis. The error represents mismatch, from which the sum of squares has been minimized by the linear regression method.

**Relation between TSR and TransDif.** In the normal heart, myofiber strain is about uniformly distributed. Myofibers run approximately circumferentially at midwall, which is defined here by equal cross-sectional areas inside and outside the midwall radius (4). Mean natural myofiber strain ($\Delta \varepsilon_{f, \text{mean}}$) is expressed as a change ($\Delta$) in

$$\varepsilon_{f, \text{mean}} = \frac{1}{2} \ln \left( 1 + 2 \frac{A_{\text{cav}}}{A_{\text{wall}}} \right)$$  \hspace{1cm} (4)

The normalized TransDif in natural myofiber strain has been quantified as

$$\text{TransDif} = \frac{\Delta \varepsilon_{f, \text{epi}} - \Delta \varepsilon_{f, \text{endo}}}{\Delta \varepsilon_{f, \text{mean}}}$$  \hspace{1cm} (5)

where $\Delta \varepsilon_{f, \text{epi}}$ and $\Delta \varepsilon_{f, \text{endo}}$, refer to the epicardial and endocardial myofiber strain, respectively. A model of LV wall mechanics has been used to express TransDif as a function of TSR and the ratio of cavity to wall area ($A_{\text{cav}}/A_{\text{wall}}$), resulting in the following approximation (APPENDIX)

$$\text{TransDif} = \left[ 1.57 - 1.16 \left( \frac{A_{\text{cav}}}{A_{\text{wall}}}, \mu \right) \right] \left( \frac{\text{TSR}}{\text{TSR}_0} - 1 \right)$$  \hspace{1cm} (6)

$\text{TSR}_0$ represents the normal value of TSR, which appeared to be $0.44 \pm 0.072$ in the control group of the present study. The subscript me refers to mid-ejection, which is the average between the begin and end of ejection (APPENDIX).

**Statistical analysis.** Differences were analyzed for significance using a two-tailed t-test, supposing unequal variances. Within the group of patients, differences between before and after aorta valve replacement were evaluated intraindividually. Values are presented as means $\pm$ SD.

**RESULTS**

Figure 2 shows representative results of analyzing the MRI-tagged images for the LV of a healthy volunteer. The top tracings show wall and cavity area for the two short-axis sections of the heart as a function of time during the heart cycle. With the use of Eq. 1, from these areas the $\varepsilon_{i\text{inner}}$ was determined. After $\sim$260 ms, $\varepsilon_{i\text{inner}}$ reaches a plateau, indicating the end of the ejection phase by closure of the aortic valve. In Fig. 2, rigid body rotations for both slices and calculated torsion (Eq. 2) are plotted as a function of time. In Fig. 3, torsion is plotted as a function of $\varepsilon_{i\text{inner}}$. The slope of the linear regression line through these data points represents the negative of the TSR value (Eq. 3).

In Fig. 4, torsion is plotted as a function of the logarithm of $\varepsilon_{i\text{inner}}$ for the control, AVSten, and AVRepl groups. The slopes of the linear regression lines reflect the individual TSR values. In Table 1, the mean values and SDs for these groups are presented for age, $A_{\text{wall}}$, $A_{\text{cav}}$, $\varepsilon_{i\text{inner}}$, torsion, TSR, and TransDif. If applicable, values are presented for the beginning and end of ejection. Both beginning and end of ejection values of $\varepsilon_{i\text{inner}}$ in the control group were larger ($P < 0.05$) and varied less than in the AVSten and AVRepl groups. Apparently, the LV of the patients was concentrically hypertrophied with no significant difference between the AVSten and AVRepl groups. In the control group, calculated myofiber shortening appears to be in a narrow range ($0.142 \pm 0.010$). In both the AVSten and AVRepl groups, myofiber shortening is less than in the control group, $P < 0.0001$ and $P < 0.02$, respectively. Myofiber shortening in the AVSten and AVRepl groups was not different as a group as well as intraindividually.

In the control group, TSR ($0.444 \pm 0.073$) was lowest with the smallest variance. In the AVSten group, the TSR value ($0.85 \pm 0.26$) was elevated ($P < 0.001$) and
variable. In the AVRepl group, the TSR value (0.61 ± 0.09) decreased (P < 0.02) but was still elevated relative to the control group (P < 0.001).

When estimating TransDif according to Eq. 5 (Fig. 5), this variable was 0.00 ± 0.14 in the control group, 0.96 ± 0.62 in the AVSten group, and 0.37 ± 0.20 in the AVRepl group. Statistical differences between the groups were very similar to those of the parameter TSR. In all patients being monitored before and after aortic valve replacement, TransDif decreased, indicating development toward a more uniform transmural distribution of myofiber shortening.

DISCUSSION

In patients, the TSR of the LV during the ejection phase was determined noninvasively using MRI tagging. In the presence of an aortic stenosis, TSR was significantly higher than in a group of healthy volunteers. After aortic valve replacement, TSR partially returned to normal. With the use of a model of cardiac mechanics (APPENDIX), the value of TSR was converted to the parameter TransDif, which expresses the epicardial-endocardial difference in myofiber shortening as normalized to mean myofiber shortening. In patients with aortic valve stenosis, TransDif was elevated relative to the control group, indicating severe contractile dysfunction of the inner layers. After aortic valve replacement, TransDif decreased to an intermediate value, indicating an important recovery of contractile function of the inner layers. MRI tagging appears the first technique by which a transmural difference in myofiber shortening can be quantified noninvasively in humans.

Patients with aortic valve stenosis have elevated end-diastolic concentricity of the LV (P < 0.01), implying that the wall is thick relative to the cavity diameter. Concentricity is the opposite of dilatation, which is quantified by εinner (Eq. 1; Table 1). Three months after valve replacement, end-diastolic concentricity was practically unchanged and still elevated (P < 0.02). Thus, after aortic valve replacement, recovery of subendocardial contractile function appears faster than regression of concentricity. Besides concentricity, in the AVSten group, the wall area was found to increase, indicating development of hypertrophy (P < 0.01), which partly recovered in the AVRepl group.

Because the biological variance of TransDif for normals was so small, for eight of nine AVSten patients the individual TransDif value was significantly higher than the value in the control group (P < 0.001), assuming a Gaussian distribution of the TransDif values in the control group. After aortic valve replacement, in only two of seven patients was this level of significance reached, whereas in the control group all individual values obviously were normal. Thus the presented MRI tagging technique seems well suited to detect subendocardial pathology in an individual patient.

No other parameter was found to be as conclusive as TransDif to distinguish individual AVSten patients from the control group. TSR is close and was able to detect seven of nine patients with P < 0.001. In the control group, calculated εf, mean as well as torsion...
Table 1. Summary of measurements

<table>
<thead>
<tr>
<th></th>
<th>Control (n = 9)</th>
<th>AVSten (n = 9)</th>
<th>AVRepl (n = 7)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean ± SD</td>
<td>Mean ± SD</td>
<td>Mean ± SD</td>
</tr>
<tr>
<td>Age, yr</td>
<td>31 ± 4</td>
<td>64 ± 9</td>
<td>68 ± 6</td>
</tr>
<tr>
<td>Awall, mm²</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Beginning of ejection</td>
<td>2,302 ± 372</td>
<td>2,859 ± 472</td>
<td>2,578 ± 360</td>
</tr>
<tr>
<td>End of ejection</td>
<td>2,289 ± 391</td>
<td>2,755 ± 493</td>
<td>2,428 ± 298</td>
</tr>
<tr>
<td>Acav, mm²</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Beginning of ejection</td>
<td>1,843 ± 327</td>
<td>1,621 ± 366</td>
<td>1,576 ± 425</td>
</tr>
<tr>
<td>End of ejection</td>
<td>1,100 ± 237</td>
<td>1,033 ± 265</td>
<td>941 ± 404</td>
</tr>
<tr>
<td>εinner, mean</td>
<td>−0.107 ± 0.084</td>
<td>−0.282* ± 0.162</td>
<td>−0.260* ± 0.318</td>
</tr>
<tr>
<td>Beginning of ejection</td>
<td>−0.365 ± 0.120</td>
<td>−0.507* ± 0.175</td>
<td>−0.517* ± 0.471</td>
</tr>
<tr>
<td>End of ejection</td>
<td>−0.006 ± 0.054</td>
<td>−0.067 ± 0.074</td>
<td>−0.067 ± 0.087</td>
</tr>
<tr>
<td>Difference</td>
<td>0.142 ± 0.0104</td>
<td>0.107† ± 0.022</td>
<td>0.117† ± 0.018</td>
</tr>
<tr>
<td>Torsion, rad</td>
<td>0.125 ± 0.050</td>
<td>0.039 ± 0.085</td>
<td>0.051 ± 0.072</td>
</tr>
<tr>
<td>Beginning of ejection</td>
<td>0.004 ± 0.007</td>
<td>0.016 ± 0.008</td>
<td>0.006 ± 0.004</td>
</tr>
<tr>
<td>End of ejection</td>
<td>0.118 ± 0.033</td>
<td>0.200 ± 0.053</td>
<td>0.165 ± 0.050</td>
</tr>
<tr>
<td>Difference</td>
<td>0.114 ± 0.034</td>
<td>0.184† ± 0.054</td>
<td>0.159* ± 0.050</td>
</tr>
<tr>
<td>TSR</td>
<td>0.444 ± 0.073</td>
<td>0.853† ± 0.264</td>
<td>0.605† ± 0.085</td>
</tr>
<tr>
<td>TransDif</td>
<td>0 ± 0.14</td>
<td>0.96† ± 0.62</td>
<td>0.37† ± 0.20</td>
</tr>
</tbody>
</table>

n = No. of healthy volunteers (control) and patients with aortic value stenosis before (AVSten) and 3 mo after aortic value replacement (AVRepl). Awall, wall area; Acav, cavity area; εinner, logarithm of normalized inner diameter; εl, mean, mean natural myocardial strain; TSR, torsion-shortening ratio; TransDif, normalized transmural differences of fiber shortening. *P < 0.05 and †P < 0.001, significantly different from the control group.

(Table 1) appeared to be in a narrow range, 0.142 ± 0.010 and 0.114 ± 0.034, respectively. In the AVSten group, these values were changed highly significantly, P < 0.0001 and P < 0.002, respectively. Nevertheless, because of the large spread in the control group, these parameters were less suited to detect individual patients. Only four and one of nine patients were found to be abnormal, respectively.

During ejection, measured motion of the LV wall caused the cross-sectional area of the wall to decrease slightly (Table 1). This decrease is likely due to an artifact in the measurement of radial displacement near the inner and outer boundaries of the wall. At those locations, the MRI tags are often malformed, introducing errors in displacement calculation. This displacement has been used to follow the contours of the wall. The method of analysis has been set up such that the influence of this error is moderate and does not affect any major conclusion of this study.

Control measurements were performed in young volunteers, whereas the patients were much older (Table 1). Current insights suggest that the TSR is independent of age and species. Assuming homogeneity of mechanical load in the heart, TSR was predicted in a mathematical model of LV mechanics (3) without using any species-specific information. TSR seems a mathematically determined constant that is expected to be universal to all hearts, “from mice to elephants.” The mathematically predicted value was found not to be different from experimental measurements in normal hearts of dogs (3) and of young volunteers (1). It has also been reported that both torsion (12) and ejection fraction in mice are not different from that in humans, suggesting similarity of TSR. Nevertheless, it is wise to perform measurements of TSR in an age-matched group of humans. Unfortunately, these data are not available yet.

The presented MRI-tagging technique is the first clinically feasible technique to quantify transmural differences in myocardial contraction. The MRI protocol takes ~25 min, finding the position and orientation of the heart inclusive. In the current setup, time is needed for selection and transfer of files. After image acquisition, epic- and endocardial contours have to be drawn manually in one set of images, which takes ~5 min. This step is the most cumbersome one and should
be automated for easy clinical application. Finally, image and data analysis take another 5 min on a regular personal computer. Various parts of the protocol and analysis may be improved, such as the use of breath-hold in MRI tagging.

In conclusion, using MRI tagging, the motion of two cross sections of the LV was measured noninvasively in healthy volunteers (control) and in AVSten and AVRepl patients. TSR in the AVSten group was significantly larger than in the control group. In the AVRepl group, TSR declined but did not return completely. With the use of a model of LV mechanics, from TSR and $A_{cav}/A_{wall}$, TransDif was calculated as normalized to mean myofiber shortening. In the control, AVSten, and AVRepl groups, TransDif was $0.62 \pm 0.14$, $0.96 \pm 0.62$, and $0.38 \pm 0.20$, respectively. Thus, in aortic valve stenosis, decreased subendocardial contractile function was found to recover 3 mo after aortic valve replacement. Over the same period, recovery of the shape of the heart (concentricity) appeared insignificant. In the AVSten group, eight of nine patients were recognized by a significant reduction of TransDif ($P < 0.001$) relative to the control group.

**APPENDIX**

**Relation between TSR and TransDif.** The value of the TSR was derived directly from the MRI tagging data. In this appendix, TSR will be related to the transmural difference in myofiber shortening as expressed by the parameter TransDif (Eq. 4). Consider the equatorial wall as a thick-walled cylinder section with inner radius $r_i$ and outer radius $r_o$. Wall coordinates $[r, c, z]$ represent the radial, circumferential, and base-to-apex directions. For small rotational symmetric deformations of an incompressible cylinder, circumferential strain $\varepsilon_{cc}$, base-to-apex strain $\varepsilon_{zz}$, and shear angle $\varepsilon_{zc}$ can be expressed as a function of radius $r$ as

$$\varepsilon_{zc}(r) = A + Br^{-2} \quad \varepsilon_{zz}(r) = 2A \quad \varepsilon_{cc}(r) = Cr$$

(7)

Parameters $A$, $B$, and $C$ are constants. To solve the problem, a sufficient number of equations has to be found. For small deformations, myofiber strain ($\varepsilon_{f}$) depends on a change in the ratio of cavity volume ($V_{cav}$) to wall volume ($V_{wall}$) (2). With the use of Eq. 7, and by substituting the inner and outer radius of the cylinder section, it is found that

$$\varepsilon_{fเฉาan} = \frac{\Delta(V_{cav}/V_{wall})}{1 + 3V_{cav}/V_{wall}} = \frac{\Delta(A_{cav}/A_{wall})}{1 + 3A_{cav}/A_{wall}} = 2B$$

$$r_o^2 + 2r_i$$

(8)

Given a myofiber angle $\alpha$, for myofiber strain it holds that

$$\varepsilon_{f}(r, \alpha) = \varepsilon_{zc} \cos^2 \alpha + \varepsilon_{zz} \sin^2 \alpha + \varepsilon_{zc} \sin \alpha \cos \alpha$$

(9)

For the normal heart, $\varepsilon_{f}$ is assumed to be the same everywhere (4). Thus three equations were obtained for the inner surface ($r = r_o$), for the midwall surface ($r = r_{mid}$), with $2r_{mid}^2 = (r_i^2 + r_o^2)$, and on the outer surface ($r = r_i$)

$$\varepsilon_{f}(r_{mid}, 0) = \varepsilon_{fเฉาan}$$

$$\varepsilon_{f}(r, \alpha_0) = \varepsilon_{fเฉaan}$$

(10)

At midwall, the myofibers are directed circumferentially (11), implying that $\alpha_{mid} = 0$. The parameter TSR is the ratio of shear at the outer surface and relative inner diameter decrease

$$\text{TSR} = \frac{\varepsilon_{zc}(r_o)}{\varepsilon_{cc}(r_i)}$$

(11)

Now assume that $r_i$, $r_o$, and TSR$_o$ are known. Equations 8, 10, and 11 then provide five equations with five unknowns, i.e., $A$, $B$, $C$, $\alpha_i$, and $\alpha_o$. This system of equations was solved using Mathematica 4.1 (Wolfram Research; Champaign, IL). As a result, the fiber angles $\alpha_i$ and $\alpha_o$ are known.

In the control group for midsection, the ratio of cavity to wall volume was $(A_{cav}/A_{wall})_o \approx 0.64$ and TSR$_o = 0.444$ (Table 1). With the use of these data, it was derived that $\alpha_i = 0.68$ rad and $\alpha_o = -0.94$ rad. Given a different value of TSR, the value of $C$ in Eq. 7 changes, causing only shear $\varepsilon_{zc}$ to change. With Eq. 9, myofiber strain was calculated numerically at the inner and outer surfaces. With Eq. 5, the parameter TransDif was calculated. For the occurring range (Table 1), the following approximation was found

$$\text{TransDif} = \left[1.20 - 1.16 \left(\frac{A_{cav}}{A_{wall}}\right)_o + 0.82 \text{TSR}_o\right] \left(\frac{\text{TSR}}{\text{TSR}_o} - 1\right)$$

(12)

By substituting $\text{TSR}_o = 0.444$, this equation results in Eq. 6.

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**REFERENCES**


