Morphometry and strain distribution of the C57BL/6 mouse aorta

X. GUO,1 Y. KONO,2 R. MATTREY,2 AND G. S. KASSAB1
1Department of Biomedical Engineering, University of California, Irvine 92697-2715; and 2Department of Radiology, University of California, San Diego, California 92103

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Guo, X., Y. Kono, R. Mattrey, and G. S. Kassab. Morphometry and strain distribution of the C57BL/6 mouse aorta. Am J Physiol Heart Circ Physiol 283: H1829–H1837, 2002.—The goal of the present study was to obtain a systematic set of data along the length of the mouse aorta to study variations of morphometry (diameter, wall thickness, and curvature), strain, and stress of the mouse aorta. Five mice were imaged with a 13-MHz ultrasound probe to determine the in vivo diameter along the aorta. A cast was made of these aortas to validate the ultrasonic diameter measurements. The root mean squared and systematic errors for these measurements were 12.6% and 6.4% of the mean ultrasound diameter, respectively. The longitudinal variations of geometry, stress, and strain from the aortic valve to the common iliac bifurcation were documented. Our results show the residual circumferential strain leads to a uniformity of transmural strain of the aorta in the loaded state along the entire length of the aorta. Furthermore, we validated the incompressibility condition along the length of the aorta. These data of normal mice will serve as a reference state for the study of disease in future knockout models.

INBRED STRAINS OF MICE are being used with increasing frequency as subjects in many experimental cardiovascular studies (7, 17). In particular, the strain C57BL/6 is perhaps the most widely used and hence the best known of all inbred strains. The popularity stems from the knockout studies that have led to substrains that have a heritable lifespan (8, 14, 25, 26, 30) and a heritable characteristic pattern of disease (6, 25, 27, 31). Despite the tremendous progress in the genotypic characterization of the mouse cardiovascular genome, however, the phenotypic characterization of the morphometry and mechanical properties of the mouse aorta is lagging behind.

The objective of the present investigation was to provide a database on the variations of morphometry along the long axis of the mouse aorta in the loaded, no-load, and zero-stress states. Specifically, we measured the cardiac output and aortic dimensions using ultrasound methodology and the longitudinal variation of the geometry of the aorta using casting methodology. The geometric data were used to compute the variations of stress and strain along the length of the aorta. The present model of normal mice will serve as a physiological reference state for future pathological studies.

METHODS

Animal Preparations

A group of 10 homozygous inbred male mice (C57BL/6 strain) were used in this study (Jackson Laboratories). The weight and age of the 10 mice were 25.3 ± 1.3 gm (range, 23.6–27.3 g) and 66 ± 1.8 days (range, 64–69 days), respectively. The mice were preanesthetized with intraperitoneal injections of ketamine (56.2 mg/kg) and midazolam (3.75 mg/kg) to achieve requisite immobilization. The mice were then anesthetized with an intraperitoneal injection of pentobarbital sodium (40 mg/kg). The mice were placed on a heating pad to maintain the body temperature during the experiment.

Ultrasound Methods

The aortas of five mice were used for an ultrasound study using a Sonoline Elegra (Siemens Ultrasound; Issaquah, WA) with a 13-MHz linear transducer. M-mode was applied vertically to the vessel, and the diameter of the aorta was measured at end-diastolic, peak systolic, and end-systolic time points at a number of anatomic sites (the ascending aorta, celiac, renal, and iliac). The peak velocity at the ascending aorta was measured using duplex Doppler. The peak cardiac output was calculated as the product of peak velocity and cross-sectional area, A (\(A = \pi D^2/4\), where \(D\) is the diameter of the aortal). At the end of the ultrasound procedure, the animals were allowed to recover.

Preparation of Casts

One day after the ultrasound procedure, each of the five animals was again anesthetized as described above. The carotid artery was cannulated for blood pressure measurement using a heat-stretched catheter prepared from MicroRenathane tubing as previously reported (21, 23). A trache-
ostomy was performed, and the mouse was ventilated with a rodent respirator. The chest was opened with a midline sternotomy. The blood was heparinized with 0.1 ml of normal saline with 200 U/ml of heparin via the carotid artery catheter. A 5-0 suture was then loosely placed around the ascending aorta. The animal was heavily sedated with pentobarbital (80 mg/kg), and the apex of the heart was excised. A 25-gauge needle was then inserted into the ascending aorta through the apex of the left ventricle. The suture was then used to ligate the catheter into the ascending aorta near the aortic valve. The blood was immediately flushed out of the aorta using a 6% dextran solution. The aorta was then perfused at 100 mmHg with catalyzed silicone elastomer containing 0.1% Cab-O-Sil (Eastman Kodak), which is a colloidal silica that forms agglomerated particles with effective diameters that exceed those of small arteries (18).

Hence, the flow of elastomer is zero, during the hardening of cast, and the pressure is 100 mmHg throughout the aorta. In addition to the five mice that underwent ultrasound imaging, five more mice were subjected to the casting protocol. All experiments were performed in accordance with national and local ethical guidelines, including the Institutional Review Board for Laboratory Animals, NIH Guide for the Care and Use of Laboratory Animals, and University of California at San Diego policies regarding the use of animals in research.

**Determination of the Loaded, No-Load, and Zero-Stress State of the Aorta**

After 45 min were allowed for the elastomer to harden, the aorta was carefully dissected and placed in a Ca²⁺-free Krebs solution composed of (in mM) 117.9 NaCl, 4.7 KCl, 1.2 MgCl₂, 25 NaHCO₃, 1.2 NaH₂PO₄, 0.0027 EDTA, 0.1 ascorbic acid, and 11 glucose. Each aorta was dissected from the aortic valve down to the common iliac artery. The aorta was then placed in Ca²⁺-free Krebs solution, and the anterior position of the aorta was marked with carbon black particles. A local dimensional coordinate, \( x \), was introduced along the loaded vessel trunk, with \( x = 0 \) at the aortic valve and \( x = 1 \) at the point of common iliac bifurcation. The external geometry of the aorta, at the loaded state, was photographed to obtain the loaded outer diameter along the trunk of the aorta. The coordinates of the centerline of the aortic arch were obtained, and the radius of curvature was computed.

The entire length of the aorta was then cut into consecutive 1-mm rings. Each ring was then transferred to Ca²⁺-free Krebs solution, aerated with 95% O₂ and 5% CO₂. The elastomer was then pushed out of each section and photographed for incompressibility condition. The first assumption implies that a small volume element (\( \delta V \)) of the vessel wall in the loaded state is equal to that in the no-load state, i.e.

\[
\delta V = A_o \delta l_o.
\]  

where \( A_o \) and \( \delta l_o \) are the wall area and length of a small vessel element at the no-load state, respectively. The second assumption leads to the following equation

\[
\delta V = \pi[(r_o^2 - r_i^2)] \delta l
\]  

where \( r_o \) and \( r_i \) are the outer and inner radius at the loaded state, respectively, and \( \delta l \) is the length of a small vessel element at the loaded state. Equations 3 and 4 can be combined to yield an expression for the inner radius as

\[
r_i = \sqrt{r_o^2 - \frac{A_o}{\pi \lambda_o}}.
\]  

where \( \lambda_o = \frac{\delta l}{\delta l_o} \) and is the stretch ratio in the axial direction. It should be noted that \( r_i \) and \( A_o \) are functions of coordinate axes \( x \) and \( \alpha \), respectively. Because \( A = x l/\lambda_o \), Eq. 5a can be rewritten as

\[
r_i(x) = \sqrt{r_o^2(x) - \frac{A_o(x)/\lambda_o(x)}{\pi \lambda_o(x)}}
\]
Equation 5b can be used to compute the longitudinal variation of inner radius along the aorta and compared with actual measurements from the cast to examine the validity of the incompressibility condition.

Mean value. The mean value of any parameter \( f(x) \) (e.g., diameter, wall thickness, area, opening angle, etc.) that varies along the length of the aorta is given by the mean value theorem as

\[
\bar{f} = \frac{1}{b-a} \int_{a}^{b} f(x) dx \tag{6a}
\]

Because our interval \([a,b]\) is defined as \([0,1]\), Eq. 6a becomes

\[
\bar{f} = \int_{0}^{1} f(x) dx \tag{6b}
\]

where \(x\) is the FLP.

Data Analysis

The position along the aorta was normalized with respect to the total length. Hence, the results were expressed in terms of the FLP, ranging from 0 to 1. The data for both the independent (FLP) and dependent variables (diameter, wall thickness, area, opening angle, etc.) were then divided into 10 equal intervals: \(0–0.1, 0.11–0.2, 0.21–0.3, \ldots 0.91–1.0\). The results are expressed as means \(\pm\) SD over each interval. Either linear or nonlinear regressions were used to curve fit the data. The longitudinal variations were examined with a one-way ANOVA. Student’s \(t\)-test was also used to detect possible differences between animals.

RESULTS

An ultrasound image of the aorta is shown in Fig. 1. The end-diastolic diameter of the aorta from five animals was measured at the ascending aorta, celiac, renal, and iliac bifurcation. Cast data were obtained from the same five animals, and a comparison of diameter measurements using the casts and the ultrasound technique is shown in Fig. 2. The relationship between the cast diameter \((D_c)\) and the ultrasound diameter \((D_u)\) can be expressed as \(D_c = 0.88 D_u + 0.17\) using a linear least-squares fit \((r^2 = 0.912)\). The root mean squared and systematic errors for these measurements were 12.6% and 6.4% of the mean ultrasound diameter, respectively. The peak systolic cardiac output and heart rate of the five mice that underwent ultrasound measurements were \(2.93 \pm 0.57\) ml/min and \(314 \pm 103\) beats/min, respectively. The average systolic/diastolic arterial blood pressure was \(107\pm 12/83\pm 9\) mmHg.

In addition to the five mice described above, five more mice were also used for the making of casts. Hence, the subsequent data obtained from cast measurements correspond to a total of 10 animals. All
Morphological and mechanical data were averaged over each of the 10 intervals and are shown as a function of FLP. The number of measurements in each interval was \(25 \pm 30\) for the 10 animals, because 25–30 measurements were made along the length of the aorta of each animal and the data were then grouped into 10 intervals.

Table 1 shows the relationship between the various anatomic bifurcations and the FLP. By definition, the aortic valve and the common iliac bifurcation are at a FLP of 0 and 1, respectively. The longitudinal position of the various bifurcations (thoracic, abdominal, renal, etc.) averaged over the 10 animals is shown in Table 1. The variations between animals are expressed through the SDs.

The lengths of the aorta, from the aortic valve to the common iliac bifurcation, were \(41.4 \pm 2.41\) and \(29.5 \pm 1.69\) mm in the loaded and no-load states, respectively. The differences in these values were statistically significant \((P < 0.0001)\). Hence, the average longitudinal stretch ratio is \(\approx 1.4\). The variation of the longitudinal stretch ratio over the length of the aorta is shown in Fig. 3. The data were fitted by a fourth-order polynomial, and the constants of the least-square fit are summarized in Fig. 3.

The variations of the loaded inner and outer diameters of the aorta are shown in Fig. 4A. The wall thickness at the loaded state was computed as the difference between the outer and inner radius of the vessel as shown in Fig. 4B. The wall thickness shows a linear relationship.

![Table 1. FLP of the aortic bifurcations](image)

<table>
<thead>
<tr>
<th>FLP</th>
<th>Aortic Branch</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.041 ± 0.0098</td>
<td>Aortic semilunar valve</td>
</tr>
<tr>
<td>0.075 ± 0.011</td>
<td>Innominate artery</td>
</tr>
<tr>
<td>0.12 ± 0.011</td>
<td>Left common carotid artery</td>
</tr>
<tr>
<td>0.20 ± 0.013</td>
<td>Left subclavian artery</td>
</tr>
<tr>
<td>0.24 ± 0.014</td>
<td>First pair of intercostal arteries</td>
</tr>
<tr>
<td>0.29 ± 0.0066</td>
<td>Second pair of intercostal arteries</td>
</tr>
<tr>
<td>0.34 ± 0.0087</td>
<td>Third pair of intercostal arteries</td>
</tr>
<tr>
<td>0.38 ± 0.0080</td>
<td>Fourth pair of intercostal arteries</td>
</tr>
<tr>
<td>0.42 ± 0.0064</td>
<td>Fifth pair of intercostal arteries</td>
</tr>
<tr>
<td>0.47 ± 0.0087</td>
<td>Sixth pair of intercostal arteries</td>
</tr>
<tr>
<td>0.51 ± 0.0096</td>
<td>Seventh pair of intercostal arteries</td>
</tr>
<tr>
<td>0.55 ± 0.012</td>
<td>Eighth pair of intercostal arteries</td>
</tr>
<tr>
<td>0.61 ± 0.012</td>
<td>Ninth pair of intercostal arteries</td>
</tr>
<tr>
<td>0.69 ± 0.013</td>
<td>Subcostal artery</td>
</tr>
<tr>
<td>0.73 ± 0.012</td>
<td>Celiac artery</td>
</tr>
<tr>
<td>0.78 ± 0.014</td>
<td>Right renal artery</td>
</tr>
<tr>
<td>0.86 ± 0.015</td>
<td>Left renal artery</td>
</tr>
<tr>
<td>0.91 ± 0.015</td>
<td>Ilium arterial</td>
</tr>
<tr>
<td>1</td>
<td>Common iliac artery</td>
</tr>
</tbody>
</table>

Values are means ± SD. FLP, fractional longitudinal position.

Fig. 3. The longitudinal stretch ratio \(\lambda_z\) as a function of the fractional longitudinal position (FLP). The solid line is a polynomial regression curve of the form \(\lambda_z = -2.0FLP^4 + 1.2FLP^3 + 1.8FLP^2 - 0.61FLP + 1.2, R^2 = 0.987\). Data correspond to means ± SD.

Fig. 4. Morphometry of the mouse aorta in the loaded state (100 mmHg). A: inner and outer diameters (\(D_i\) and \(D_o\), respectively) as a function of FLP. Least-square curves are of the fourth-order polynomials \(D_i = 9.1FLP^4 - 20.2FLP^3 + 14.6FLP^2 - 4.5FLP + 1.7, R^2 = 0.989\), and \(D_o = 8.9FLP^4 - 19.7FLP^3 + 14.3FLP^2 - 4.4FLP + 1.6, R^2 = 0.989\). B: wall thickness (WT). A least-square fit is used to fit the wall thickness data as \(WT = -26.8FLP + 56.5 (R^2 = 0.998)\). C: radius of curvature (RC) of the centerline can be described by a least-square fit as \(RC = 805FLP^2 - 180FLP^2 + 12.4, R^2 = 0.995\). Data correspond to means ± SD.
variation whose empirical constants are summarized in Fig. 4. The thickness-to-radius ratio varies from 0.066 to 0.085 from the aortic valve to the common iliac bifurcation, respectively. The radius of curvature of the centerline of the aortic arch is shown in Fig. 4. Figure 5, A and B, shows the longitudinal variation of the wall thickness and wall area for the aorta in the no-load state.

The open sector in the zero-stress state was characterized by the opening angle, which is defined as the angle subtended by two radii connecting the midpoint of the inner wall. The longitudinal variation of the opening angle is shown in Fig. 6A for the aorta. The variation of inner and outer residual strain, over the length of the aorta, was not found to be statistically significant, as shown in Fig. 6B. The mean values of the inner and outer residual strains over the length of the aorta were $-0.055 \pm 0.012$ and $0.058 \pm 0.012$, respectively. The negative strains imply residual compression at the inner wall, whereas the positive strains imply residual tension at the outer wall (see the Appendix). At the loaded state, both the inner and outer strains become positive, as shown in Fig. 6C. The variation of inner and outer strain was not found to be statistically significant with mean values of $0.39 \pm 0.14$ and $0.42 \pm 0.13$ for the loaded inner and outer strains, respectively. The differences were not statistically significant.

A comparison between the measured inner radius from the cast and that computed from the incompressibility condition (Eq. 5) is shown in Fig. 7. The average circumferential and longitudinal stresses were computed according to Eq. 1. The variations of stresses, along the length of the aorta, were not found to be statistically significant with mean values of $155 \pm 20.6$ and $74.4 \pm 3.86$ kPa for the circumferential and longitudinal stresses, respectively.
DISCUSSION

Validation of Ultrasonic Measurements

We measured the dimensions of the aorta at several positions along the long axis of the aorta as identified by anatomic bifurcations (e.g., ascending aorta, celiac, renal, and iliac bifurcation). The same locations in the same mice were measured using a casting method. The results are shown in Fig. 2 with the root mean squared and systematic errors of 12.6% and 6.4% of the mean ultrasound diameter, respectively. Previously, Chiou et al. (2) measured the diameters of the mouse aorta and inferior vena cava with the transrectal ultrasound (20 MHz) technique and then with direct visualization upon laparotomy. Their data showed a percent error of 13.7–14.2%. Hence, our method provides similar accuracy to the higher frequency transrectal method.

Cardiac Output and Anesthesia

The values of peak systolic cardiac output measured in the ascending aorta were lower than those reported by Krivitski et al. (19), which ranged from an average of 6.8 to 12.7 ml/min. The anesthetic agents (ketamine, midazolam, and pentobarbital) used slowed heart rate significantly. Resting heart rate for the conscious mouse has been reported at >500 beats/min, but heart rate is substantially lower under anesthesia (21). At present, the in vivo technique is limited to the study of anesthetized mice because the mice must be immobile for the study. Because the cardiac output is proportional to the heart rate, a reduction in heart rate will cause a reduction in cardiac output. In future studies, we will investigate other anesthetic agents that may have fewer effects on heart rate and hence cardiac output.

Longitudinal Morphology of the Aorta

The FLP of the aorta relative to the various anatomic bifurcations is documented in Table 1. These data serve to quantify the anatomy of the aorta relative to its bifurcations. The data are averaged over 10 mice. The variability of the bifurcation positions along the aorta can be described by the coefficient of variance, CV (CV = SD/mean), at each FLP. It can be seen that the CV is much larger proximal to the thoracic aorta. These are important considerations when designing experiments on various segments of the aorta.

The longitudinal variation of the diameter of the aorta is shown in Fig. 4A. It should be noted that the taper geometry is only an approximation. In reality, the diameter changes take place only at the branching points, i.e., the vessel segments are cylindrical between bifurcation points. This observation was first made by Sobin and Chen (29) and applies to many vascular and botanical trees. Sobin’s law postulates that the apparent taper of a large artery is due to side branches. Hence, the taper is only an approximation to, what is in reality, a staircase change in diameter.

The variation of wall thickness is linear in the loaded and no-load states, as shown in Figs. 4B and 5A, respectively. Again, it should be noted that the taper geometry is only an approximation, as discussed above. The slope of the taper is $-6.5 \times 10^{-4}$ and $-1.1 \times 10^{-3}$ in the loaded and no-load states, respectively. A linear variation has also been previously found in the pig aorta in the no-load state (15). The taper slope can be estimated as $-4.6 \times 10^{-3}$ for the pig aorta in the no-load state. No data were obtained in the loaded state for the pig aorta. The variation in the wall area is, however, nonlinear and can be described by a fourth-order polynomial, as shown in Fig. 5B. Han and Fung (16) previously determined the longitudinal variation of the wall cross-sectional area of the dog and pig aorta. Again, they found species differences in the thoracic region. The similarity between the longitudinal variation of the wall area of the pig and mouse aortas is remarkable.

The variation of the radius of curvature of the aortic arch is shown in Fig. 4C. It can be seen that the curvature (inverse of radius of curvature) is greatest at a FLP of 0.11. An analysis of stress distribution in the aortic arch must take into account the curvature because larger curvature leads to a greater nonuniformity in the circumferential stress distribution. Our data are similar to those found for the aortic arch of the rat (20).

Longitudinal Variation of the Zero-Stress State

It has been nearly 20 years now since Vaishnav and Vossoughi (32) and Fung (10) independently showed that the zero-stress state of a blood vessel is an open sector. This was a critical observation for vessel mechanics because all computations of stress and strain must be referred to the zero-stress state, which had erroneously been assumed to be the no-load configuration (i.e., a tube). Since then, numerous papers have been written on the subject (for a review, see Ref. 11).

The longitudinal variation of the opening angle has been previously documented in the rat and pig aortas (15, 20). Both pig and rat aortas have a mean opening angle of $\sim 130^\circ$ in the ascending aorta region similar to that found in the mouse. The species differences be-
come evident, however, in the thoracic aorta region. The opening angle of the pig aorta in the middle thoracic region is fairly uniform with a value of $\sim 60^\circ$ and increases to a mean value of $\sim 80^\circ$ near the iliac bifurcation. The rat, however, shows considerable variation, decreasing to a mean value of $10^\circ$ (including some negative angles) at the lower end of the thoracic region near the diaphragm and increases to a mean value of $90^\circ$ near the iliac bifurcation. The spatial complexity in the rat was not observed in our mouse model. In fact, we never observed negative opening angles along the entire length of the aorta, as shown in Fig. 6A. We found a mean value of $97^\circ$ in the thoracic aorta, which increased to $134^\circ$ near the iliac bifurcation. It appears that the aortic longitudinal variation of the opening angle in our mouse model is more similar to the pig than to the rat. Furthermore, the variation of opening angle in the present study was similar to postmortem data of the human aorta, which did not show a minimum value in the middle portion of the aorta (28). Direct numeric comparison of opening angles is difficult because the human aorta data were obtained 24 h after death.

Uniform Transmural Strain Hypothesis

Fung and Liu (12) have previously shown that one of the implications of the existence of the residual strain is to make the transmural strain distribution more uniform. This was demonstrated in ileal, medial planar, and pulmonary arteries of the rat. In the present study, we tested this hypothesis along the entire length of the aorta. Our results show that the negative inner and positive outer residual strains (Fig. 6B) lead to a uniform transmural strain distribution in the loaded state throughout the length of the aorta (Fig. 6C). This uniformity of transmural strain has important physiological implications (12).

Incompressibility Condition

The incompressibility condition has been previously shown to be a reasonable assumption for blood vessels under physiological conditions (1, 4). In the present study, the incompressibility condition, Eq. 5, was used to compute the inner diameter of the vessel based on measurements of the outer diameter (Fig. 4A), no-load wall area (Fig. 5B), and longitudinal stretch ratio (Fig. 3). Because we directly measured the inner diameter using silicone elastomer casts, we can compare our experimental measurements with the theoretical predictions, as shown in Fig. 7. It is apparent that the agreement is excellent with root mean squared and systemic errors of 6.9% and $-4.9\%$ of the mean measured aortic radius, respectively. Hence, the mouse aorta is incompressible. This is an important finding, so that in future studies, when only the inner or outer dimension is measured, the other can be accurately computed according to Eq. 5.

Significance of the Study

The mouse has become a very popular model for transgenic manipulation. Hypertension, hypercholesteremia, and diabetes have been well established in this animal model, and all are known to be important risk factors for the development of cardiovascular disease. With the establishment of a model of aortic morphology and strain distribution in normal mice, we can determine the effect of different risk factors in future studies.

APPENDIX

Analysis of the Stresses and Residual Stresses in the Arterial Wall

To compute residual stress from the residual strain in an organ, one must first be able to analyze the stress and strain distribution in the organ under a load. The following example illustrates these concepts.

An Idealized Example

Consider a cylindrical segment of an unloaded artery that, when cut longitudinally, open up into a circular sector with an opening angle $\theta$, an inner radius $a$, and an outer radius $b$. We will compute the residual stress in the vessel when it is unloaded and uncut. To solve this problem, we assume that the arterial wall obeys Hooke's law of linear elasticity. For a two-dimensional, plane stress problem, the solution can be expressed in terms of an Airy stress function that is the result of the integration of the differential equations of equilibrium together with the compatibility equations [see Fung (9)]. The nonvanishing components of the stress (circumferential ($\tau_{\theta\theta}$) and radial ($\tau_{rr}$), respectively) are given as follows

\begin{equation}
\tau_{\theta\theta} = -\frac{A}{r} + B(3 + \ln r^2) + 2C \tag{7a}
\end{equation}

\begin{equation}
\tau_{rr} = \frac{A}{r^2} + B(1 + \ln r^2) + 2C \tag{7b}
\end{equation}

where $r$ is the radius of the vessel and $A, B,$ and $C$ are constants of integration and depend on the boundary conditions. The circumferential displacement for the circular sector ($\xi_\theta$) is given by

\begin{equation}
\xi_\theta = \frac{8\pi Br}{E} \tag{8}
\end{equation}

where $E$ is the Young's modulus of elasticity. The circumferential displacement required to deform the sector into a circle is given by $\xi_\theta = r \sin \theta$. If we combine this expression with Eq. 8 and assume a small opening angle (so that sin $\theta \approx \theta$), we can solve for the constant $B$ in terms of $\theta$ and $E$ (i.e., $B = \theta E/8\pi$). The constants $A$ and $C$ can be determined by imposing the boundary conditions

\begin{equation}
\tau_{rr} = -P \quad \text{at} \quad r = a \tag{9a}
\end{equation}

\begin{equation}
\tau_{rr} = 0 \quad \text{at} \quad r = b \tag{9b}
\end{equation}

After some algebraic manipulation, Eqs. 7b and 9 yield a solution for the constants $A$ and $C$ as
If we consider \( \theta = \pi/4 \) [i.e., \( \sin(\theta/\omega) \approx 0.9 \)] and reduce the pressure to zero, we obtain the distribution of circumferential stress in the no-load state (residual stress), as shown in Fig. 8A. Please note that the circumferential stress is negative at the inner wall and positive at the outer wall. This is consistent with the measured residual strains. Figure 8B shows the radial distribution of circumferential stress at 100 mmHg. Please note that in the case where \( \theta = 0 \), we recover Lamé's solution, i.e.

\[
\tau_{\theta\theta} = P \left( \frac{b}{r} \right)^2 + 1
\]

If we integrate Eq. 11 through the wall thickness, we obtain the average circumferential stress as given by Eq. 1a, i.e., Laplace's equation.

\[
A = \left[ \frac{1}{1 - \left( \frac{1}{a^2} \right)} \right] \left[ P - \frac{\alpha E}{8\pi} \ln \left( \frac{b^2}{a^2} \right) \right]
\]

\[
C = -\frac{1}{2} \left[ 1 - \left( \frac{1}{b^2} \right) \right] \left[ P - \frac{\alpha E}{8\pi} \ln \left( \frac{b^2}{a^2} \right) \right] - \frac{\alpha E}{16\pi} \left( 1 + \ln b^2 \right)
\]

Extension to a More Realistic Approach

The constitutive equation of the arterial wall is nonlinear. Chuong and Fung (3) have shown that the strain-energy function for the arterial wall can be represented by the equation

\[
\rho_0 W = c \exp (\alpha_i E_i E_j)
\]

where \( W \) is the pseudostress energy per unit mass, \( \rho_0 \) is the mass density, \( c \) and \( \alpha_i \) (where \( ij = 1, 2, 3 \) and are constants, and \( E_1, E_2, \) and \( E_3 \) are the principal strains defined in the sense of Green. The partial derivative of \( \rho_0 W \) with respect to \( E_i \) is the principal stress defined in the sense of Kirchhoff. Patel and Vaishnav (22) prefer to express \( \rho_0 W \) as a polynomial in \( E_i \), of order 3 or higher, which results in equally good fitting with experimental results. With a nonlinear constitutive equation, the solution of the stress distribution in arteries is more complex. Chuong and Fung (4) reduced the artery problem to two integral equations and obtained the solution numerically.

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