Influence of structural geometry on the severity of bicuspid aortic stenosis

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Richards, Kathryn E., Dimitri Deserranno, Erwan Donal, Neil L. Greenberg, James D. Thomas, and Mario J. Garcia. Influence of structural geometry on the severity of bicuspid aortic stenosis. Am J Physiol Heart Circ Physiol 287: H1410–H1416, 2004.—Doppler-derived gradients may overestimate total pressure loss in degenerative and prosthetic aortic valve stenosis (AS) due to unaccounted pressure recovery distal to the orifice. However, in congenitally bicuspid valves, jet eccentricity may result in a higher anatomic-to-effective orifice contraction ratio, resulting in an increased pressure loss at the valve and a reduced pressure recovery distal to the orifice leading to greater functional severity. The objective of our study was to determine the impact of local geometry on the total versus Doppler-derived pressure loss and therefore the assessed severity of the stenosis in bicuspid valves. On the basis of clinically obtained measurements, two- and three-dimensional computer simulations were created with various local geometries by altering the diameters of the left ventricular outflow tract (LVOT; 1.8–3.0 cm), orifice diameter (OD; 0.8–1.6 cm), and aortic root diameter (AR; 3.0–5.4 cm). Jet eccentricity was altered in the models from 0 to 25°. Simulations were performed under steady-flow conditions. Axisymmetric simulations indicate that the overall differences in pressure recovery were minor for variations in LVOT diameter (<3%). However, both OD and AR had a significant impact on pressure recovery (6–20%), with greatest recovery being the larger OD and the smaller recovery being the AR. In addition, three-dimensional data illustrate a greater pressure loss for eccentric jets with the same orifice area, thus increasing functional severity. In conclusion, jet eccentricity results in greater pressure loss in bicuspid valve AS due to reduced effective orifice area. Functional severity may also be enhanced by larger aortic roots, commonly occurring in these patients, leading to reduced pressure recovery. Thus, for the same anatomic orifice area, functional severity is greater in bicuspid than in degenerative tricuspid AS.

Doppler; echocardiography; congenital heart disease

CONGENITAL BICUSPID AORTIC VALVE is the most common cause of isolated aortic stenosis (AS) (5, 16, 17), the effects of which typically appear clinically in the adult population. Doppler-derived gradients are routinely used in clinical practice to determine the severity of the stenosis with the presumption that pressure drop across the valve is irretrievably lost (12). However, studies (7) have shown that in certain geometric situations (most commonly bileaflet mechanical prostheses), a significant amount of pressure loss is recovered distal to the valve orifice so that the Doppler-derived gradient overestimates the net pressure drop. It has been suggested that pressure recovery in AS depends on the ratio of the valve orifice area and the cross sectional area of the aorta, for this particular ratio determines the extent of flow separation and the formation of vortexes (4, 10, 13). Several researchers (1, 9) have demonstrated that pressure recovery is more prominent in less severe AS, especially when the orifice area is >0.7 cm². A unifying characteristic of pressure recovery is early reattachment of flow to the wall of the valve or aorta (15). Total pressure recovery is dependent on geometry and the quantity of energy lost due to friction. Therefore, pressure recovery may be reduced in bicuspid aortic valve because as the eccentric jet hits the wall and energy is lost to heat, flow separation, and vortex formation (1, 3, 4, 6, 11). Thus functional severity may be relatively greater in bicuspid than in degenerative trileaflet valves for the same anatomic valve area. In addition, eccentric orifices have a reduced effective flow area. Therefore, larger velocities are required to sustain the same flow rate. These larger velocities could also contribute to the increased pressure loss at the level of the orifice.

The purpose of this study was to use computational fluid dynamics modeling to investigate the effect of geometric factors [size of left ventricular outflow tract (LVOT), orifice area, and aortic root (AR)] and jet eccentricity on the orifice pressure gradients to address the discrepancy between Doppler and catheter measurements.

METHODS

Clinical data. To determine the ranges for the parameters to build our model, we reviewed the transthoracic and transesophageal echocardiographic (TEE) examinations of 50 consecutive patients (28 male, age 56±14 yr) with either congenitally bicuspid or degenerative AS who were referred for both conditions between January 2000 and December 2001. An institutional review board approved this project. Transthoracic examinations were performed on all patients using HDI-5000 (Philips; Bothel, WA), Sequoia (Siemens; Mountain View, CA), or Sonos 5500 (Philips; Andover, MA) ultrasound machines. All TEE exams were performed using a multiplane transducer after local oral anesthesia with viscous lidocaine and light intravenous sedation with midazolam and fentanyl. With the use of probe anteflexion and multiplane rotation between 35° and 60°, short-axis views of the aortic valve were performed to obtain a cross-section of the valve with complete visualization of the leaflet edges during peak systole. Adequate manipulation of the probe was done to obtain the smallest aortic valve cross-sectional area during systole, which was then measured by planimetry. The average of three valve orifice diameter (OD) measurements was used for analysis. To ascertain that the measurements corresponded to the leaflet edges, the vertical diameter of the orifice was compared with the same diameter obtained form the long axis (120–150°) view. The TEE examination was also used to measure the LVOT diameter 1–1.5 cm below the valve and the AR diameter 1 cm distal to the sinotubular junction, using the average of three measurements. The AR was measured distally because this is more representative of the average diameter of the aortic root. All
measurements were performed off-line from the digitally stored trans-thoracic and TEE images (ProSolv; Indianapolis, IN). The angle between the AR axis and the stenotic color Doppler jet was measured on exported Bitmap images using Osiris software (Digital Imaging Unit, University Hospital; Geneva, Switzerland).

**Concentric valvular geometry model.** A rigid axisymmetric fluid dynamics model of the aortic valve was produced in a computational fluid dynamics software package (Fluent version 5.3, Fluent; Lebanon, NH) based on the measurements obtained from the clinical dataset. A total of 24 axisymmetric models with simplified geometry was developed, each with ~32,000–48,000 elements. An axisymmetric model could be used because we assumed that the LVOT, OD, and AR had a circular shape, under concentric jet conditions. The anatomic oriﬁce area varied between 0.5–2.0 cm² and 0.8–1.6 cm OD. The LVOT diameter ranged from 1.8 to 3.0 cm and the AR diameter varied from 3.0 to 5.4 cm. An example of the mesh near the oriﬁce is shown in Fig. 1.

Simulations were conducted on each modeled geometry under steady flow conditions. Although inlet pressure varies with time in reality, a quasi-steady state could be assumed at peak pressure because at this time the rate of developed pressure over time is zero. The simulations had inlet total pressure ranging from 30–70 mmHg. The outlet static pressure was set at 0 mmHg. A total of 72 simulations were conducted with these parameters on the various geometries. Each simulation took ~30 min. Centerline pressure profiles, velocity contour plots, and mass flow rates were recorded. In addition, a series of simulations were performed on various geometries with different oriﬁce areas in which the mass flow rate was ﬁxed, so that the overall pressure drop could be observed. The ﬁxed inlet mass ﬂow rate was 0.35 kg/s, corresponding to a stroke volume of 55 ml. Each simulation was conducted with blood analog ﬂuid settings, including density of 1,050 kg/m³ and the viscous level of 0.003 Pa/s. The turbulence model used was the standard k-ε model. Wall properties were set at standard no-slip conditions.

**Three-dimensional eccentric valvular geometry model.** Similar to the previous section, an additional set of geometries was created to allow eccentric jets. With the use of a computational ﬂuid dynamics software package (Fluent 5.3), a rigid three-dimensional (3-D) symmetric model of the aortic valve was developed, allowing for the investigation of the impact of jet eccentricity on oriﬁce pressure gradients. A computational mesh including ~120,000 hexahedral elements was produced for each simulated geometry. To better represent the anatomic conﬁguration of bicuspid aortic stenosis, the inlet geometry was slightly adjusted, as shown in Fig. 2. Similar limitations apply to these models, with the exception that the 3-D geometry was symmetric about a plane bisecting the aorta, not axisymmetric, allowing eccentric jets through a circular oriﬁce. On the basis of the data obtained from the clinical studies, the AR diameter was set with a range of 3.0–5.4 cm and the LVOT diameter with a range of 1.8–3.0 cm. In addition, clinical studies indicate that bicuspid aortic stenosis versus tricuspid valve stenosis are characterized by increased jet eccentricity (mean angles of 33° vs. 9°, respectively) (Table 1).

A total of 15 geometries was created to simulate degenerative and congenital bicuspid AS with an oriﬁce plane that varied to produce a jet with 0–25° of eccentricity relative to the aortic axis. Previous literature (3) suggested that angles >25° would demonstrate little difference.

For each geometry, simulations were performed under steady flow conditions with ﬁxed mass ﬂow rates. The range of mass ﬂow rate investigated was determined from physiological ranges and 33 simulations were conducted with the mass ﬂow rate ranging from 39–63 ml/beat corresponding to 0.1412–0.24 kg/s on the symmetric inlets. The ﬂow rate used is half of the normal ﬂow rate because the simulation is technically half of the total system. Outlets were left as outﬂow zones. Walls were set with a no-slip boundary condition. Turbulence levels were set at standard k-ε values. The fluid properties

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**Fig. 1.** Mesh of the axisymmetric model.
were the same as in the axisymmetric models. The overall pressure drop and velocity contour plots were recorded. Each 3-D simulation required ~20 h of computing time, significantly higher than the average simulation time of 30 min in the two-dimensional (2-D) models.

RESULTS

**Concentric valvular geometry model.** The 2-D simulations provide the data for the graph in Fig. 3 that demonstrates the influence of the LVOT diameter on the pressure drop of the system over a given distance. Pressure profiles were normalized to the inlet pressure. The orifice is located at 4 cm, which explains the sudden decrease in pressure. As seen in Fig. 3, the overall differences in pressure recoveries are minor (<3%), as are the overall differences in the mass flow rates (<7%). The lack of pressure recovery differences is due to no significant changes in the velocity or pressure profile for the different LVOT diameters, as shown in the contour plot in Fig. 3, top left. The contour plot demonstrates the left-to-right movement of the fluid. At 4 cm, the fluid converges to fit through the orifice creating a central jet. On exiting the orifice, the jet spreads to fill the aortic root that in turn creates the pressure recovery. However, as shown, vortexes and stagnant fluid appear, minimizing the chance for ideal pressure recovery. This particular contour plot shows no difference in vortex area or velocity distal to the orifice, indicating that the pressure recovery in the two simulations would be close.

The OD, however, had a greater impact on the overall pressure recovery in the system, as shown in Fig. 4. Pressures are again normalized to the pressure at the inlet and a significant drop appears ~4 cm where the orifice is located. Unlike the differences in the LVOT diameter, the OD produced significant changes in the mass flow rate as well as significant changes in the pressure recovery. An OD of 1.6 cm had the largest mass flow rate, 0.577 kg/s, and the largest pressure recovery, 20%. An OD of 0.8 cm produced the lowest pressure recovery (6%) and the lowest flow rate (77% lower than the flow rate for OD = 1.6 cm). As shown in Fig. 4, the smaller the OD, the more pressure is lost to friction and flow acceleration before the orifice, when the fluid has to increase its velocity to get through the smaller orifice. In the aortic root some of the remaining pressure was lost in the vortex that in turn decreases the chance for pressure recovery. Therefore, with smaller OD, vortexes and other frictional forces waste more energy resulting in reduced availability of pressure for flow acceleration at

Table 1. *Jet eccentricity in tricuspid versus bicuspid aortic stenosis*

<table>
<thead>
<tr>
<th></th>
<th>n</th>
<th>AVAc, cm²</th>
<th>α, °</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tricuspid valve</td>
<td>22</td>
<td>1.13±0.46</td>
<td>9.0±7.7</td>
</tr>
<tr>
<td>Bicuspid valve</td>
<td>49</td>
<td>1.36±0.77</td>
<td>33.4±9.7*</td>
</tr>
</tbody>
</table>

Values are means ± SE; n, no. of subjects. AVAc, effective valve area; α, eccentricity angle. *P < 0.01.
the orifice, rendering smaller peak velocities and mass flow rates for a given overall pressure gradient.

Figure 5 demonstrates the influence of the AR diameter on pressure gradients. Again, the pressure was normalized to inlet conditions. However, after the pressure drop at the orifice (Graph A), the pressure profiles differed because of the difference in AR size. For the small AR diameter (3.0 cm), the pressure recovery and the mass flow rate were greatest (20% and 339 ml/s, respectively). For the large AR diameter (5.4 cm), the lowest mass flow rate and degree of pressure recovery were observed. The pressure recovery was significantly less (6%) while the mass flow rate was only 8% less (315 ml/s). The model with AR = 5.4 cm had a large vortex that extended all along the wall, whereas AR = 3.0 cm resulted in a small vortex using less energy. The increased energy losses in large aortic roots lead to a reduced availability of pressure for flow acceleration at the orifice, rendering smaller peak velocities and mass flow rates for a given overall pressure gradient.

To describe overall pressure loss, a pressure loss coefficient ($K_L$) was defined based on the static pressure drop across the valve and the maximum dynamic pressure, as described by Eq. 1:

$$P_{prox} - P_{dist} = K_L \frac{1}{2} \rho v_{max}^2$$

where $P_{prox}$ is proximal pressure, $P_{dist}$ is distal pressure, $\rho$ is viscosity, and $v_{max}^2$ is maximum continuous Doppler velocity. As pressure loss increases with increasing LVOT and AR, and decreases with increasing OD, a ratio describing the constriction and expansion geometry were considered in evaluating $K_L$. Defining a constriction ratio (CR) as LVOT/OD and an expansion ratio (ER) as AR/OD, we found through linear regression analysis that $K_L$ has a strong correlation with $1/(CR^{0.5} \cdot ER)$.

Figure 6 demonstrates the relationship between $K_L$ and this geometric factor for the 72 axisymmetric 2-D simulations. This best-fit relationship has a correlation value of 0.93 and is described by the following formula:
\[ K_L = 1 - \frac{2.63}{(CR \cdot ER)^2} \]  

(2)

A 3-D eccentric valvular geometry model. The impact of jet eccentricity on pressure gradients investigated through 3-D models is given in Fig. 7. This graph illustrates the changes in pressure loss for different flow rate and jet eccentricities for constant CR and ER. The graph demonstrates as expected that with these particular geometries the higher the mass flow rate (and subsequently the peak velocity), the larger the pressure drop overall. Looking at a simulation of a 4° versus a 24° eccentricity more closely in Fig. 8, the following can be observed: the 4° jet is slightly off center and creates a slight downward jet with little to no reduction in effective orifice flow area, whereas, in the 24° geometry, the downward jet has significantly reduced effective orifice area. These increased peak velocities required a larger pressure gradient required for flow acceleration as shown in the pressure profiles of Fig. 8. In addition, the increased vortex on the top wall also slightly reduced pressure recovery in the 24° model (8.6% vs. 10.7%).

Figure 9 demonstrates the impact of CR and ER on overall pressure drop for a constant flow rate. Again, the typical increase in change in pressure is observed with greater eccentricity. Although all of the lines have similar behavior, an interesting difference occurs in the eccentric valves in relation to the LVOT. The 2-D simulations were influenced little by the CR due to simplified local orifice geometry (see Fig. 1). However, the CR had a much larger impact on the eccentric jets due to the more realistic local orifice geometry in the 3-D model, including proximal LVOT tapering (see Fig. 2). The ER had similar impact on the eccentric simulations as in the 2-D simulations. In summary, the larger the ER or CR, the larger was the pressure loss, in agreement with Eq. 2.

From the computer simulations (Fig. 10), one can see that in a larger ER or a larger CR there is more chance of a vortex formation, which causes a loss in energy that could have been transferred into pressure recovery. The simulations with larger ER show that there is a greater vortex formation after the valve than in the smaller ER, leading to a larger change in pressure, as is seen in Fig. 9. For instance, the pressure loss from the ratio of 2:2.5 is 16% less than the pressure loss acquired by the 2:4.5 in the 14° simulation with the same flow rate and orifice area. Likewise, for the larger CR, there is more pressure loss because there is a larger vortex before the valve. For instance, the ratio of 2.5:3.5 has a 20% greater pressure loss than the 1.5:3.5 ratio. These data state that the smaller the LVOT and AR, the greater the chance of reduced overall pressure gradient in eccentric bicuspid valves. In addition, larger orifice areas and centric jets will further reduce pressure losses.

Therefore, patients with bicuspid aortic valves, which exhibit greater aortic root dilatation and eccentric jets, will have a greater functional severity for the same orifice area than patients with tricuspid aortic stenosis.

With the use of the same definition for \( K_L \), a new formula was obtained to include the effects of jet eccentricity. A similar structure is used as in the previous formula of \( K_L \); however, an additional term was added, including the jet eccentricity angle, \( \alpha \). Linear regression analysis shows that Eq. 3 has a correlation value of 0.91

\[ K_L = 0.93 \left[ 1 - \frac{5}{(ER \cdot CR)^2} \right] \left[ 1 + \left( \frac{\alpha}{90} \right)^2 \right] \]  

(3)

where \( \alpha \) is jet eccentricity angle.
DISCUSSION

The results of our numeric simulation demonstrate that various geometric factors result in variable pressure gradient at the orifice and recovery distal to the stenotic orifice in AS. Our findings were in agreement with those of previous investigators (3, 10, 13), demonstrating that smaller AR dimension along with larger orifice areas increases the chance of reduced pressure gradients. Our results also demonstrate that jet eccentricity also plays an important role, increasing the pressure gradients at the orifice and limiting pressure recovery and therefore increasing functional severity in cases of bicuspid AS. These results are in agreement with the findings of VanAuker et al. (14). The 3-D simulations demonstrated the effect of orifice angles on various geometries with different contraction and expansion ratios. In summary, the smaller the eccentricity, the smaller the AR or LVOT, and the greater the orifice area, the smaller the gradients and the functional severity of the AS.

Because Doppler techniques do not take recovery into consideration, they may overestimate functional severity (7). The magnitude of pressure recovery increases as the AR diameter-to-OD ratio approaches 1. This may occur in some clinical situations, such as moderate stenosis in patients with small ARs. Discrepancies that exist between Doppler ultrasound and catheter gradients are mainly due to pressure recovery or the rise in static pressure downstream from a stenotic aortic valve (2, 8). When a catheter is placed downstream from a stenosed valve, it will measure a relatively accurate recovered pressure gradient, but because it cannot reach the vena contracta or maximum velocity point with great accuracy, it tends to underestimate the maximum pressure drop across the orifice (2, 3, 7, 8). This in contrast to Doppler gradients, which are based on peak flow velocities of the stenotic orifice.

Table 2 illustrates the expected absolute and relative differences between Doppler and catheter gradients and its dependence on LVOT diameter, anatomic OD, AR diameter, and jet eccentricity. Four cases of constant flow rate are considered (371 ml/s). In the first case, LVOT diameter was varied around the mean (2.4 cm), whereas OD (1.2 cm), AR (4.2 cm), and eccentricity (0°) were kept constant, similarly for cases 2–4. From Table 2, it is clear how a LVOT diameter increases leads to an increase of the pressure drop (both Doppler and catheter). However, as LVOT increases in size, the pressure recovery decreases reflected by a reduced Doppler-catheter difference. Next, as expected, smaller ODs lead to increased pressure losses and peak velocities across the valve in both catheter and Doppler measurements. In addition, a larger anatomic OD results in a smaller absolute (5 mmHg) but greater relative (41%) difference between catheter and Doppler gradients. With smaller peak velocities, there is a greater chance for full pressure recovery, explaining the higher relative Doppler-catheter differences, although the absolute difference may be minor due to the low overall pressure drop. Whereas AR diameter does not significantly impact peak velocities and Doppler gradients, an increased AR leads to reduced pressure recovery.

Table 2. Catheter versus Doppler-derived gradients

<table>
<thead>
<tr>
<th>Dimensions, cm or °</th>
<th>Peak Velocity, m/s</th>
<th>Doppler, mmHg</th>
<th>Catheter, mmHg</th>
<th>Difference, mmHg</th>
<th>Difference, %</th>
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</thead>
<tbody>
<tr>
<td>LVOT diameter</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.8</td>
<td>3.61</td>
<td>52.1</td>
<td>35.6</td>
<td>16.5</td>
<td>46.4</td>
</tr>
<tr>
<td>3</td>
<td>3.71</td>
<td>55.1</td>
<td>48.5</td>
<td>6.6</td>
<td>13.5</td>
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<td>Orifice diameter</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>0.8</td>
<td>8.49</td>
<td>288.3</td>
<td>274.4</td>
<td>13.9</td>
<td>5.1</td>
</tr>
<tr>
<td>1.6</td>
<td>2.17</td>
<td>18.8</td>
<td>13.4</td>
<td>5.4</td>
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</tr>
<tr>
<td>Root diameter</td>
<td></td>
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<td></td>
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</tr>
<tr>
<td>3</td>
<td>3.76</td>
<td>56.6</td>
<td>42.3</td>
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<tr>
<td>5.4</td>
<td>3.74</td>
<td>56.0</td>
<td>49.8</td>
<td>6.2</td>
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<td>Eccentricity</td>
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<tr>
<td>0</td>
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<td>47.6</td>
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<td>24</td>
<td>4.27</td>
<td>72.9</td>
<td>60.8</td>
<td>12.1</td>
<td>20.0</td>
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</table>

LVOT, left ventricular outflow tract.
recovery, reflected by higher catheter gradients and Doppler-catheter differences. Finally, with the eccentric valves, an increase in the peak velocity and pressure drop (both catheter and Doppler) across the valve is observed with increases jet eccentricity, as illustrated in Fig. 7. However, whereas the absolute pressure drop increases with increasing eccentricity, the relative difference between Doppler and catheter gradients remains fairly constant.

Despite these considerations, clinical studies have reported excellent agreement between catheter and Doppler-derived gradients in AS. One possible explanation for this better than predicted agreement observed in the clinical practice is that most patients with AS have poststenotic dilation of the root. Also, catheter gradients may also overestimate the true pressure loss if the distal measurement is taken too close to the orifice because pressure recovery may continue to occur between 5–10 cm distally.

Finally, to illustrate the clinical applicability of the model, the following example is considered. In the computational model, an average simulated anatomic valve area of 1.1 cm² demonstrated an average effective orifice area of 0.8 cm². In a preliminary clinical study performed in 49 patients with bicuspid AS, a significant relationship between anatomic and effective valve area has been found ($y = 0.69x + 0.09; r = 0.88; n = 50; P < 0.01$). With the use of the clinical obtained regression, a 1.1-cm² anatomic area corresponds to a 0.85-cm² effective area, similar to the effective area from the fluid dynamics model.

**Limitations.** The model consists of rigid geometries, not considering the elastic properties of the real material in the vessel walls; however, because we used steady flow, an elastic simulation would be irrelevant because the walls would not contract or expand. Our simulations were conducted in ideal simplified geometry neglecting the aortic sinuses and any further malformation that may occur in aortic stenosis. However, our simplified geometry of the stenosis does provide very useful information.

Whereas axisymmetrical conditions (circular LVOT, orifice, AR, and a circular central jet) were employed in the 2-D simulations, the results from such simulations matched closely to similar geometries solved in 3-D space. Asymmetric geometry would affect our findings if the asymmetry contributes to jet direction and vortex size. Both phenomena lead to increased pressure losses due to increased amount of energy absorbed in the vortices and a decreased effective flow area. In any case, the asymmetry would provide results similar to reduced orifice area and/or increased root diameter.

Also, the use of steady flow neglects the acceleration through the valve for these particular cases. The results should reflect events with good accuracy at the peak flow during ejection. At this point, the pressure gradient across the valve is consumed in full by the resistance of the valve. However, during other phases of ejection, these results will not be accurate because the pressure drop across the valve contributes to both pressure losses and flow acceleration (or deceleration) over time, the latter being absent in a steady-state model. Therefore, future work could include the effects of unsteady flow conditions with more realistic conditions, such as detailed geometry and elastic material properties.

**REFERENCES**