Left ventricular regional wall curvedness and wall stress in patients with ischemic dilated cardiomyopathy

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ISCHEMIC DILATED CARDIOMYOPATHY (IDCM) is a degenerative disease of the myocardial tissue accompanied by left ventricular (LV) remodeling (34). LV remodeling is a multistep process that involves acute dilation of the infarcted area, increase of LV volume, lengthening of the LV perimeter, and decrease of LV curvature (10, 42). Natural history studies show that progressive LV remodeling is directly related to future deterioration of LV performance and a poor clinical course (20).

The elevation of LV wall stress in IDCM is associated with morphological changes in the myocardium that may cause regional hypokinesis (20–22). LV wall stress is in part determined by the local curvature of the ventricular wall; i.e., decreased curvature will increase wall stress (23). In addition to increasing LV size, IDCM can alter myocardial properties and normal LV shape curvature. The border zone will have a higher stress, which makes it more susceptible to ischemia and infarction and may accelerate the remodeling process (44). Therefore, therapeutic approaches for IDCM include LV size reduction to disrupt the downward-spiral cycle of heart failure (12).

The geometry of the LV in IDCM is complicated, and a three-dimensional (3-D) approach is necessary to characterize the ventricular mechanics of contractility and regional shape changes throughout the cardiac cycle. Many studies have shown that MRI is a better and more comprehensive approach to quantify 3-D ventricular structure and function (5, 28) than echocardiography (17), ventriculography (16, 48), angiography (40), and indicator-dilution (11) methods.

End-systolic wall stress is governed by the systolic pressure of the LV and a geometric factor of the shape of the LV (19, 23, 47). Several mathematical models have been used to calculate wall stress (47). Some approaches are limited by geometric assumptions and are only valid for spherical or ellipsoidal shapes and, hence, only allow calculation of global wall stress. Other approaches are applicable to any arbitrary shape of the LV (23) but have some errors in the measurement of the thickness of the LV wall and of the radius of curvature (6). The technical limitations of these methods do not permit precise regional measurements of 3-D wall curvature.

In this study, we aim to 1) assess the regional variations of LV shape in 3-D (i.e., in terms of surface curvedness), 2) assess the 3-D regional variations of wall stress (by incorporation of wall curvature), and 3) determine the relationship between peak systolic wall stress and the regional extent of myocardial infarct.

Glossary

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>IDCM</td>
<td>Ischemic dilated cardiomyopathy</td>
</tr>
<tr>
<td>dσ*/dt&lt;sub&gt;max&lt;/sub&gt;</td>
<td>LV contractility index [(1.5 \times \frac{dV}{dt_{\text{max}}})/(V_m)]</td>
</tr>
<tr>
<td>EDVI</td>
<td>Indexed end-diastolic volume</td>
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METHODS

Subjects. Ten normal subjects and 10 patients with IDCM participated in the study. All subjects underwent diagnostic MRI scan. None of the normal subjects had 1) significant valvular or congenital cardiac disease, 2) history of myocardial infarction, 3) coronary artery lesions, or 4) abnormal LV pressure, end-diastolic volume, or ejection fraction. All subjects were recruited without consideration of gender or ethnicity and gave informed consent. The study was approved by the Human Subjects Review Committee of the National Heart Centre, Singapore.

MRI scans. MRI scanning was performed using steady-state free precession cine gradient echo sequences. Subjects were imaged on a 1.5-T scanner (Avanto, Siemens Medical Solutions, Erlangen, Germany). Some preliminary short-axis acquisitions were used to locate the plane passing through the mitral and aortic valves. This allowed the acquisition of the oblique long-axis plane of the LV, which is orthogonal to the short-axis plane and passes through the mitral valve, apex, and aortic valve. In addition, the corresponding vertical long-axis planes (connecting the short-axis plane images) were acquired. TrueFISP (fast imaging with steady-state precession) magnetic resonance pulse sequence with segmented k-space and retrospective electrocardiographic gating was used to acquire a parallel stack of two-dimensional (2-D) cine images of the LV in the short-axis plane, from LV base to apex (8-mm interslice thickness, no interslice gap). The field of view was typically 320 mm with in-plane spatial resolution of <1.5 mm. Each slice was acquired in a single breath hold, with 25 temporal phases per heart cycle. Figure 1 depicts the magnetic resonance short- and long-axis plane or image during end diastole and end systole. The average duration of entire image acquisition period was 30 min. The short- and long-axis views (or planes) derived from the MRI were utilized to carry out 3-D LV reconstruction (at end diastole and end systole) using customized software (see Data processing and LV geometry reconstruction).

Delayed gadolinium-enhancement imaging. For evaluation of myocardial viability, 10 to 15 contiguous short-axis views were acquired with 2-D MRI. Standard extracellular magnetic resonance contrast agents were injected (0.2 mmol gadolinium/kg iv, gadoterate dimeglumine; Schering, Berlin, Germany). All images were acquired during end-expiratory breath holds at 5–15 min after injection. T1 weighting was achieved with an inversion-recovery fast low-angle shot (IR-FLASH) pulse sequence. Typical parameters were as follows: TE = 2 ms, TR = 6 ms, and voxel size = 1 x 1 x 6 mm, with 300-ms inversion delay and k-space data segmented over four cardiac cycles (32 lines per cycle) and data acquired every cardiac cycle. Images were acquired at mid-diastole during breath hold (8 s).

Using commercially available image analysis software (Syngo 3D, Siemens Medical Solution), one author (R.-S. Tan, with more than 10 years of experience in cardiac magnetic imaging) designated the regions of interest in the viable myocardium (dark) and in the nonviable myocardium (highly enhanced) for each image frame in the short-axis cine DE series. The thickness of the hyperenhancement was planimetered on all short-axis images from base to apex. Each 3-D reconstructed LV model was divided into 16 segments (see Fig. 4, Tables 2–4).
myocardial contrast in each of these segments was quantified in terms of 
scar scores. A scar extent score was assigned to each region as 
follows: 0 = no scarring, 1 = 1–25% scarring, 2 = 26–50% scarring, 
3 = 51–75% scarring, 4 = 76–99% scarring, and 5 = 100% scarring. 

Data processing and LV geometry reconstruction. The MRI data 
were processed using a semiautomatic technique provided in the 
CMRtools suite (CVIS, London, UK). Short- and long-axis images 
were displayed simultaneously, such that segmentation in the two 
planes proceeded interactively to reduce registration errors (35). For 
each phase, every control point on the endocardium was constrained 
to lie on the intersection of the short- and long-axis views. 

The construction of long-axis planes (orthogonal to the short-axis 
planes oriented at regular angular intervals) enabled the fitting of a 
series of B-spline curves to represent the contours of the endocardial 
surface. This allowed the addition or manipulation of control points to 
obtain the desired boundary locations. The papillary muscles and 
trabeculae were included in the chamber volume to obtain smooth 
endocardial contours suitable for shape analysis. The 3-D reconstruc-
tions of a typical IDCM and normal LV during end-diastolic and 
end-systolic phases are shown in Fig. 2. 

From the reconstructed LV, the chamber volume, wall mass, stroke 
volumes, and EF were calculated. A six-order polynomial function 
was then used to curve fit the volume-time data to determine the 
volume rate (dV/dt). The contractility index (dF/dtmax) was calcu-
lated using the following formula: 1.5 × (dV/dtmax)/Vm, where Vm is 
myocardial volume at the end-diastolic phase, as previously reported 
(49). Global LV shape was assessed by calculation of the sphericity 
index (SI) in diastole using the following formula: SI = AP/BA, 
where BA was measured in the four-chamber view from the apex to 
the midpoint of the mitral valve and AP was measured as the axis that 
perpendicularly intersects the midpoint of the long axis. A small SI 
value implies an ellipsoidal LV, whereas values approaching 1 sug-
gest a more spherical LV. 

Computation of 3-D surface shape descriptors. Software developed 
in-house was used to reconstruct the LV endocardial meshes at end 
diastole and end systole as well as to calculate the surface shape 
descriptors, expressed in terms of local normal curvature and curved-
ness. The formulations for these descriptors are shown in APPENDIX A. 

To evaluate the shape at a particular point on the mesh, a local 
surface geometry was fitted to the region. The normal curvature at that 
point is then calculated as 

\[ \kappa(\lambda) = \frac{L + 2M\lambda + N\lambda^2}{E + 2F\lambda + G\lambda^2} \]  

where \( \lambda = \frac{dv}{du} \), such that \( u \) and \( v \) are the parameters of the 
underlying geometry, and \{E, F, G\} and \{L, M, N\} are components of 
the first and second fundamental forms, respectively. 

Fig. 2. Three-dimensional (3-D) reconstruction of LV en-
docardial surface at end diastole and end systole for an 
ischemic dilated cardiomyopathy (IDCM) patient (A) and a 
normal subject (B).
Innovative Methodology

LEFT VENTRICULAR SHAPE

The extreme values \( \kappa_1 \) and \( \kappa_2 \) of \( \kappa(\lambda) \) are the maximum and minimum principal curvatures, respectively, and they are obtained from the roots of the equation

\[
\det \begin{bmatrix}
L - \kappa E & M - \kappa F \\
M - \kappa F & N - \kappa G
\end{bmatrix} = 0
\]

(2)

The corresponding directions of \( \kappa_1 \) and \( \kappa_2 \) are the principal directions and are orthogonal to each other. Figure 3 presents typical principal curvature on the endocardial wall of the LV in IDCM and normal hearts.

A shape descriptor is characterized in terms of the curvedness value \( C \), as presented by Koenderink and Van Doorn (25). It is a measure of the degree of regional curvature and is defined as

\[
C = \sqrt{\frac{\kappa_1^2 + \kappa_2^2}{2}}
\]

(3)

The value of \( C \) indicates the magnitude of the curvedness at a point, which is a measure of the extent to which a region deviates from flatness. A normal LV will exhibit a larger value of \( C \).

The percent curvedness change (\( \Delta C \)) between the end-diastolic and end-systolic phase is defined as

\[
\Delta C = \frac{C_{ES} - C_{ED}}{C_{ES}} \times 100\%
\]

(4)

where \( C_{ES} \) and \( C_{ED} \) represent curvedness at the end-diastolic and end-systolic phase, respectively. Positive values of \( \Delta C \) indicate LV wall regions of increasing curvature during systole; negative values of \( \Delta C \) indicate wall regions of decreasing curvature.

Regional wall thickening. In addition to the evaluation of curvature measures on each point of the surface mesh, the LV radius (\( R \)) and wall thickness (\( T \)) were deduced from the 3-D geometry of the LV. The steps of the computation of wall radius-to-curvature and thickness are summarized in APPENDIX B. The wall thickening (WT) is expressed at each segment by the following formula

\[
WT = \frac{EST - EDT}{EDT}
\]

(5)

where EST and EDT are wall thickness at end systole and end diastole, respectively.

Regional peak systolic wall stress. The wall stress is obtained by the equilibrium of forces due to stresses in the wall and blood pressure acting on the wall. Following Grossman et al. (19), the regional peak systolic wall stress (WS) was determined from the inner radius of curvature (\( R \)) and \( T \) at end systole by

\[
WS = 0.133 \times SP \times \frac{R}{2T \left(1 + \frac{T}{2R}\right)}
\]

(6)

where SP is the peak systolic ventricular blood pressure (in mmHg). In this study, SP was assessed from the systolic noninvasive blood pressure (37a); a conversion factor of 0.133 was used to express the final results in 1,000 N/m². WS was calculated using \( R \) and \( T \) values derived with the 3-D curvature method described above. WS was determined using Eq. 6 and as illustrated in Fig. B1 in the circumferential and longitudinal (or meridional) regions in 2-D as 2DSWS and 2DLWS and for a 3-D element with radius that is the inverse of curvature (computed from Eq. 3).

To determine regional LV properties of curvedness, wall stress, and wall thickening, the LV was divided into a 16-segment model (8) from apex to base (Fig. 4).

Statistical analysis. LV volume, function, SI, and curvedness data were compared between IDCM patients and normal subjects using a t-test. 2-D short-axis plane, long-axis plane, and 3-D evaluations of wall stress were compared using ANOVA. If there was a significant interaction (\( P < 0.05 \)) between multiple measurements, selected pairwise comparisons were examined further. The difference in curvedness, wall stress, and wall thickening among various zones was assessed by ANOVA. Statistical significance for comparison of regional LV systolic wall stress between IDCM patients and normal subjects was determined using a two-tailed Student’s t-test. A similar analysis was made for wall thickening calculated in the 2-D short-axis plane, 2-D long-axis plane, and direction of the 3-D curvature. Bivariate correlation was performed between curvedness difference, wall stress, and wall thickening and extent of myocardial infarction.
using Spearman’s correlation coefficient. Values are means ± SD, and significance was defined as \( P < 0.05 \).

**RESULTS**

*Global LV function.* The hemodynamic and volumetric parameters of the subjects are summarized in Table 1. The cardiac contractility index \( \frac{d\sigma}{dV_{\text{max}}} \) and LV ejection fraction (LVEF) were significantly lower in IDC patients than normal subjects. In addition, LV end-diastolic and end-systolic volumes were greater in IDC patients than normal subjects. By visual inspection, the LV has a broader apex in IDC patients than normal subjects. The increase in dilated volume in the LV with IDC was accompanied by a corresponding increase in sphericity. Consequently, the LV was significantly more spherical in IDC patients than normal subjects.

**Variation of curvedness, peak systolic wall stress, and wall thickening from base to apex in normal subjects.** Calculated regional values for curvedness at end diastole and end systole in normal subjects and IDC patients are summarized in Table 2. In general, normal hearts demonstrated the following regional differences: curvedness is highest at the apex and higher in the inferior regions than in the lateral region (among the 4 circumferential zones), especially at end diastole. In IDC patients, curvedness was highest at the apex, whereas there was no significant difference among the six circumferential zones. Similar to normal subjects, the gradient from base to apex was significant (\( P < 0.001 \), ANOVA).

Wall thickness and radius of the cavity were measured in the short-axis plane, long-axis plane (perpendicular to the short axis), and 3-D surface. Wall stress calculated with these data is shown in Fig. 5, A–C. Peak systolic wall stress in the short-axis plane (Fig. 5A) and long-axis plane (Fig. 5B) revealed a significant difference from base to apex (\( P < 0.0001 \), ANOVA). The short- and long-axis wall stress showed 41 ± 11% and 45 ± 12% increase of peak systolic wall stress between basal and apical sections, respectively. When wall thickness and radius of the cavity were calculated in the 3-D space, the variation of wall stress (3DWS) from base to apex was no longer observed (Fig. 5C). The difference between 2DSWS and 3DWS, 2DLWS, and 3DWS was reduced more at the base. The 3DWS values tend to be highest in the anterior region and lowest in the inferior region (Table 3).

Wall thickening values determined in the short-axis plane, long-axis plane, and 3-D surface are shown in Fig. 5, D–F. Wall thickening did not differ significantly from base to apex. The comparison between 3DWT and a 2-D assessment of wall thickening (i.e., 2DSWT and 2DLWT) did not reveal significant differences at the basal and midzone, anterior, septal, and lateral regions.

**Curvedness, radius-to-thickness ratio, peak systolic wall stress, and wall thickening in IDC patients and normal subjects.** Regional variations of LV curvature are highlighted by curvedness values from base to apex in normal subjects and IDC patients in Table 2. Significant differences in end-diastolic curvedness (C_Ed) were noted in all regions, except the base and anterior. Also, significant differences in end-systolic curvedness (C_Es) and ΔC were noted in all regions between normal subjects and IDC patients. Significant differences in end-diastolic radius-to-thickness ratio (R/T_Ed) were noted in 9 of 16 segments. Also, significant differences in end-systolic radius-to-thickness ratio (R/T_Es) were noted in all regions between normal subjects and IDC patients (Table 4).

In IDC patients, 3DWS was significantly increased and 3DWT was decreased compared with normal subjects (Fig. 6A). There is a gradient of mean wall stress from base to apex with 3DWS. Also, 3DWS was highest at the apex in IDC patients and 3DWT was significantly decreased in all regions compared with normal subjects (Fig. 6B). In addition, 3DWT is smallest at the inferior segments and highest at the anterior segments. There is also significant variation among the four circumferential regions (\( P < 0.001 \), ANOVA) and from base to apex (\( P < 0.001 \), ANOVA).

**Analysis of segment scar extent in IDC patients.** The distribution of mean values of wall stress and wall thickening with the extent of segment scar is shown in Fig. 7 for a total of 160 segments from the IDC patients. As expected, there was a positive correlation between the extent of myocardial infarct and peak systolic wall stress (3DWS; \( r = 0.652, P < 0.0001 \); Fig. 7A). A negative correlation between the extent of myo-

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**Table 1. Characteristics of normal subjects and IDC patients**

<table>
<thead>
<tr>
<th></th>
<th>Normal (n = 10)</th>
<th>IDC (n = 10)</th>
<th>( P )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age, yr</td>
<td>39 ± 17</td>
<td>52 ± 9</td>
<td>0.05</td>
</tr>
<tr>
<td>Weight, kg</td>
<td>67 ± 15</td>
<td>71 ± 16</td>
<td>0.57</td>
</tr>
<tr>
<td>Height, cm</td>
<td>169 ± 8</td>
<td>164 ± 8</td>
<td>0.18</td>
</tr>
<tr>
<td>Diastolic press, mmHg</td>
<td>73 ± 12</td>
<td>70 ± 9</td>
<td>0.54</td>
</tr>
<tr>
<td>Systolic press, mmHg</td>
<td>122 ±17</td>
<td>113 ± 12</td>
<td>0.19</td>
</tr>
<tr>
<td>HR, beats/min</td>
<td>70 ± 9</td>
<td>81 ± 18</td>
<td>0.10</td>
</tr>
<tr>
<td>CI, ml/m²</td>
<td>3.3 ± 0.4</td>
<td>2.3 ± 0.4</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>EDVI, ml/m²</td>
<td>73 ± 10</td>
<td>144 ± 27</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>ESVI, ml/m²</td>
<td>26 ± 6</td>
<td>114 ± 32</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>EF, %</td>
<td>65 ± 5</td>
<td>22 ± 9</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>SI</td>
<td>0.52 ± 0.06</td>
<td>0.62 ± 0.08</td>
<td>&lt;0.05</td>
</tr>
<tr>
<td>( \frac{d\sigma}{dV_{\text{max}}} )</td>
<td>( 5.7 ± 1.3 )</td>
<td>2.4 ± 0.9</td>
<td>&lt;0.001</td>
</tr>
</tbody>
</table>

Values are means ± SD. IDC, ischemic dilated cardiomyopathy; HR, heart rate; CI, cardiac index; EDVI, end-diastolic volume index; ESVI, end-systolic volume index; EF, ejection fraction; SI, sphericity index; \( \frac{d\sigma}{dV_{\text{max}}} \), cardiac contractility index \((=1.5 \times \frac{dV/dV_{\text{max}}}{V_m})\), where \( dV/dV_{\text{max}} \) is maximum volume rate and \( V_m \) is myocardial volume.

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**Table 2. Characteristics of normal subjects and IDC patients**

<table>
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<tr>
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</thead>
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<td>0.57</td>
</tr>
<tr>
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<td>169 ± 8</td>
<td>164 ± 8</td>
<td>0.18</td>
</tr>
<tr>
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<td>70 ± 9</td>
<td>0.54</td>
</tr>
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<td>Systolic press, mmHg</td>
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<td>113 ± 12</td>
<td>0.19</td>
</tr>
<tr>
<td>HR, beats/min</td>
<td>70 ± 9</td>
<td>81 ± 18</td>
<td>0.10</td>
</tr>
<tr>
<td>CI, ml/m²</td>
<td>3.3 ± 0.4</td>
<td>2.3 ± 0.4</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>EDVI, ml/m²</td>
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<td>144 ± 27</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>ESVI, ml/m²</td>
<td>26 ± 6</td>
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<td>&lt;0.001</td>
</tr>
<tr>
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<td>65 ± 5</td>
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<td>( \frac{d\sigma}{dV_{\text{max}}} )</td>
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cardiac infarct and wall thickening (3DWT) can be seen ($r = -0.622$, $P < 0.0001$; Fig. 7B).

**DISCUSSION**

This study investigated the regional 3-D shape variations (e.g., in terms of curvedness) and wall stress in the LV. Accordingly, we developed methods to determine and map the local curvatures, local radius, and wall thickness in a 3-D model of the reconstructed LV. This approach yields new insights and demonstrates the potential of using 3-D regional analysis to provide details associated with local mechanics that are unattainable with 2-D analysis or simplified geometric modeling.

![Table 2. LV regional curvedness analysis in normal subjects and IDCM patients](image)

<table>
<thead>
<tr>
<th>Segment</th>
<th>Normal (n = 10)</th>
<th>IDCM (n = 10)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$C_{ES}, \text{mm}^{-1}$</td>
<td>$C_{ES}, \text{mm}^{-1}$</td>
</tr>
<tr>
<td>1. Basal anterior</td>
<td>0.040±0.0098</td>
<td>0.054±0.0064</td>
</tr>
<tr>
<td>2. Basal anterior septal</td>
<td>0.037±0.0058</td>
<td>0.058±0.010</td>
</tr>
<tr>
<td>3. Basal inferior septal</td>
<td>0.030±0.0036</td>
<td>0.047±0.0062</td>
</tr>
<tr>
<td>4. Basal inferior</td>
<td>0.038±0.0055</td>
<td>0.056±0.0079</td>
</tr>
<tr>
<td>5. Basal inferior lateral</td>
<td>0.035±0.0040</td>
<td>0.055±0.0096</td>
</tr>
<tr>
<td>6. Basal anterior lateral</td>
<td>0.036±0.0063</td>
<td>0.050±0.0065</td>
</tr>
<tr>
<td>7. Middle anterior</td>
<td>0.034±0.0035</td>
<td>0.057±0.010</td>
</tr>
<tr>
<td>8. Middle anterior septal</td>
<td>0.043±0.0057</td>
<td>0.070±0.016</td>
</tr>
<tr>
<td>9. Middle inferior septal</td>
<td>0.037±0.0059</td>
<td>0.056±0.0096</td>
</tr>
<tr>
<td>10. Middle inferior</td>
<td>0.043±0.0044</td>
<td>0.060±0.010</td>
</tr>
<tr>
<td>11. Middle inferior lateral</td>
<td>0.037±0.0025</td>
<td>0.063±0.011</td>
</tr>
<tr>
<td>12. Middle anterior lateral</td>
<td>0.033±0.0029</td>
<td>0.052±0.011</td>
</tr>
<tr>
<td>13. Apical anterior</td>
<td>0.043±0.0061</td>
<td>0.083±0.016</td>
</tr>
<tr>
<td>14. Apical septal</td>
<td>0.052±0.0092</td>
<td>0.10±0.024</td>
</tr>
<tr>
<td>15. Apical inferior</td>
<td>0.056±0.0077</td>
<td>0.10±0.030</td>
</tr>
<tr>
<td>16. Apical lateral</td>
<td>0.048±0.0091</td>
<td>0.10±0.026</td>
</tr>
</tbody>
</table>

Values are means ± SD. LV, left ventricular; $C_{ES}$, curvedness in end-systole; $C_{ES}$, curvedness in end-systole; $\Delta C$, curvedness change between end diastole and end systole. *$P < 0.05$; †$P < 0.01$; ‡$P < 0.001$ vs. normal.
Normal hearts. The radius of curvature of the LV is usually estimated from the short- and long-axis diameters (2-D analysis). These parameters, however, do not necessarily correspond to the radii of curvature at a particular point, especially when local pathology is present (45). In the present study, we have compared the difference between 3-D and 2-D analysis (i.e., short axis and long axis). The wall stress values derived from short- and long-axis methods tend to vary from base to apex, with the smallest wall stress value at the apex. 3DWS is fairly uniform, however, from base to apex (P = 0.298, ANOVA) among the four circumferential regions (P > 0.05, ANOVA). Also, 3DWS is ~10% lower at the apex and 20% lower at the inferior region than elsewhere. These modest variations in wall stress are in agreement with using the relative uniform radius-to-thickness ratio from base to apex (Table 4). Previous studies of wall stress at the apex are controversial, with some predicting high values (36) and others predicting low values (3, 28).

ICDM. LV remodeling in IDC M is a multistep process that has been investigated in numerous studies (9, 34, 39). The loss of contractile function following coronary occlusion is accompanied by acute dilatation of the infarction area, increase of LV volume, lengthening of the LV perimeter, and blunting of the normal curvature. Geometrically, the increase of LV volume and sphericity result in a corresponding increase of the local radii of curvature. This is also consistent with the increase of SI, which is the short axis-to-long axis ratio. The development of IDC M is accompanied by a decrease of LV function (e.g., LVEF, do*/dmax, and wall thickening), on the one hand, and a progressive increase of LV wall stress, on the other. In the present study, we find that wall stress increases in each region, which may in turn cause decreased function (wall thickening). The increase of global stress has been shown to be a measure of afterload following infarction (47). However, information regarding regional distribution of wall stress is lacking. Since

### Table 3. LV end-systolic wall stress that takes into account curvature in normal heart

<table>
<thead>
<tr>
<th>Segment</th>
<th>3DWS, ( \times 1000 ) N/m²</th>
<th>2DLWS, ( \times 1000 ) N/m²</th>
<th>2DSWS, ( \times 1000 ) N/m²</th>
<th>3DWS vs. 2DSWS</th>
<th>3DWS vs. 2DLWS</th>
<th>2DSWS vs. 2DLWS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Basal anterior</td>
<td>10.9±3.92</td>
<td>8.19±3.55</td>
<td>10.1±4.99</td>
<td>0.01</td>
<td>0.030</td>
<td>NS</td>
</tr>
<tr>
<td>2. Basal anterior sepal</td>
<td>10.1±3.96</td>
<td>7.99±4.38</td>
<td>8.81±2.02</td>
<td>0.006</td>
<td>NS</td>
<td>NS</td>
</tr>
<tr>
<td>3. Basal inferior sepal</td>
<td>12.4±3.21</td>
<td>12.1±5.56</td>
<td>7.54±2.81</td>
<td>NS</td>
<td>&lt;0.0001</td>
<td>NS</td>
</tr>
<tr>
<td>4. Basal inferior</td>
<td>10.3±4.07</td>
<td>8.00±2.67</td>
<td>8.08±3.26</td>
<td>0.009</td>
<td>NS</td>
<td>NS</td>
</tr>
<tr>
<td>5. Basal inferior lateral</td>
<td>10.4±4.69</td>
<td>7.80±3.46</td>
<td>8.73±3.83</td>
<td>0.003</td>
<td>0.021</td>
<td>NS</td>
</tr>
<tr>
<td>6. Basal anterior lateral</td>
<td>12.5±5.50</td>
<td>9.50±4.68</td>
<td>9.39±5.80</td>
<td>&lt;0.001</td>
<td>0.003</td>
<td>NS</td>
</tr>
<tr>
<td>7. Middle anterior</td>
<td>13.7±4.52</td>
<td>9.10±3.88</td>
<td>10.0±3.93</td>
<td>&lt;0.0001</td>
<td>&lt;0.0001</td>
<td>NS</td>
</tr>
<tr>
<td>8. Middle anterior sepal</td>
<td>10.2±3.85</td>
<td>7.41±4.02</td>
<td>9.85±3.33</td>
<td>0.024</td>
<td>NS</td>
<td>0.048</td>
</tr>
<tr>
<td>9. Middle inferior sepal</td>
<td>11.9±4.69</td>
<td>10.5±6.60</td>
<td>7.41±3.44</td>
<td>NS</td>
<td>&lt;0.0001</td>
<td>NS</td>
</tr>
<tr>
<td>10. Middle inferior</td>
<td>11.1±5.77</td>
<td>8.35±3.91</td>
<td>8.05±5.74</td>
<td>0.039</td>
<td>&lt;0.0001</td>
<td>NS</td>
</tr>
<tr>
<td>11. Middle inferior lateral</td>
<td>12.6±9.63</td>
<td>8.00±5.89</td>
<td>10.8±9.49</td>
<td>0.021</td>
<td>&lt;0.0001</td>
<td>NS</td>
</tr>
<tr>
<td>12. Middle anterior lateral</td>
<td>17.3±11.12</td>
<td>12.4±9.20</td>
<td>10.2±7.31</td>
<td>&lt;0.0001</td>
<td>0.006</td>
<td>NS</td>
</tr>
<tr>
<td>13. Apical anterior</td>
<td>12.5±6.02</td>
<td>6.27±3.38</td>
<td>7.15±3.49</td>
<td>&lt;0.0001</td>
<td>&lt;0.0001</td>
<td>NS</td>
</tr>
<tr>
<td>14. Apical septal</td>
<td>8.50±4.23</td>
<td>6.07±3.79</td>
<td>5.53±2.54</td>
<td>NS</td>
<td>0.015</td>
<td>NS</td>
</tr>
<tr>
<td>15. Apical inferior</td>
<td>10.0±6.40</td>
<td>7.16±4.13</td>
<td>5.08±3.88</td>
<td>NS</td>
<td>0.003</td>
<td>NS</td>
</tr>
<tr>
<td>16. Apical lateral</td>
<td>10.7±7.40</td>
<td>5.87±3.14</td>
<td>10.7±7.40</td>
<td>0.045</td>
<td>0.006</td>
<td>NS</td>
</tr>
</tbody>
</table>

Values are means ± SD. 3DWS, 3-dimensional (3-D) wall stress; 2DLWS, wall stress in long-axis plane; 2DSWS, wall stress in short-axis plane; NS, not significant.

### Table 4. LV radius-to-thickness ratio that takes into account 3-D curvature in normal subjects and IDC M patients

<table>
<thead>
<tr>
<th>Segment</th>
<th>Normal (n = 10)</th>
<th>IDC M (n = 10)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RT/ED</td>
<td>RT/ES</td>
</tr>
<tr>
<td>1. Basal anterior</td>
<td>5.04±1.35</td>
<td>1.86±0.65</td>
</tr>
<tr>
<td>2. Basal anterior sepal</td>
<td>4.22±1.26</td>
<td>1.66±0.58</td>
</tr>
<tr>
<td>3. Basal inferior sepal</td>
<td>4.59±1.06</td>
<td>2.01±0.53</td>
</tr>
<tr>
<td>4. Basal inferior</td>
<td>3.90±0.60</td>
<td>1.72±0.61</td>
</tr>
<tr>
<td>5. Basal inferior lateral</td>
<td>4.95±1.19</td>
<td>1.84±0.80</td>
</tr>
<tr>
<td>6. Basal anterior lateral</td>
<td>5.65±1.83</td>
<td>2.15±0.96</td>
</tr>
<tr>
<td>7. Middle anterior</td>
<td>6.98±3.17</td>
<td>2.10±0.70</td>
</tr>
<tr>
<td>8. Middle anterior sepal</td>
<td>4.77±1.20</td>
<td>1.65±0.55</td>
</tr>
<tr>
<td>9. Middle inferior sepal</td>
<td>4.74±1.25</td>
<td>1.94±0.69</td>
</tr>
<tr>
<td>10. Middle inferior</td>
<td>4.18±0.83</td>
<td>1.83±0.84</td>
</tr>
<tr>
<td>11. Middle inferior lateral</td>
<td>5.15±1.00</td>
<td>2.03±1.34</td>
</tr>
<tr>
<td>12. Middle anterior lateral</td>
<td>6.26±1.87</td>
<td>2.57±1.53</td>
</tr>
<tr>
<td>13. Apical anterior</td>
<td>5.86±1.67</td>
<td>1.97±0.86</td>
</tr>
<tr>
<td>14. Apical septal</td>
<td>4.55±0.83</td>
<td>1.41±0.59</td>
</tr>
<tr>
<td>15. Apical inferior</td>
<td>4.78±1.15</td>
<td>1.59±0.89</td>
</tr>
<tr>
<td>16. Apical lateral</td>
<td>5.67±2.42</td>
<td>1.72±1.01</td>
</tr>
</tbody>
</table>

Values are means ± SD. RT/ED, end-diastolic radius-to-thickness ratio; RT/ES, end-systolic radius-to-thickness ratio. *P < 0.05; †P < 0.01; ‡P < 0.001 vs. normal.
infarction results in regional inhomogeneity of wall stress and the regional load may vary between different zones of an LV, the response to the load should be examined regionally.

Measurement of regional LV curvature. The importance of shape characteristics of ventricular chambers has been appreciated for many centuries. Much of the previous work, however, has been of a qualitative nature or based on assumptions regarding ideal geometry of the ventricle. Furthermore, many earlier studies are based on 2-D models of short and/or long axis from ventriculography (4, 20, 30, 31, 33), MRI (3), and echocardiography (29).

The present study draws on the strength of 3-D geometric methodology, which uses an analytic approach to extract local differential properties and curvature information by means of local surface fitting (14, 18, 24, 50). This approach is proven to be robust and produces accurate results, as shown by Garimella and Swartz (15) and Cazals and Pouget (8). The approach is also quantitative and provides specific regional information, and it can be extended to the characterization of 3-D LV wall stress.

The choice of the descriptor used to quantify the variation of the surface shape is important. The common shape-related differential properties of surfaces are the Gaussian curvature \( K = k_1 k_2 \) and the mean curvature \( H = (k_1 + k_2)/2 \). The Gaussian curvature is considered the most widely used index of surface shape, and it depends only on the intrinsic geometry of the surface. However, it does not intuitively describe the extent of surface bending. For example, in Fig. 8, the Gaussian curvature \( K \) at the parabolic line on a toroidal surface is zero, even though it is actually curved. On the other hand, the mean curvature \( H \) can also be misleading. For example, at the saddle point illustrated in Fig. 8, \( H \) is zero, even though the surface is curved. Koenderink and van Doorn (25) also showed that \( K \) is not very indicative of local shape, and they introduced a more significant measure of local shape known as curvedness \( C \).

The curvedness value \( C \) describes the magnitude of the curvature at a surface point, i.e., a measure of degree of curvature at a point. Since we are interested in the magnitude or extent of curvature locally, we chose \( C \) as the curvature metric for analyzing LV regional shape. Our results show that surface curvedness \( C_{ES} \) of the LV of an IDCM patient is significantly lower than \( C_{ES} \) of the normal LV. This is explicitly demonstrated by the lower systolic function (i.e., LVEF and wall thickening).

Limitations of the study. The peak systolic blood pressure only provides a global value of stress for the whole ventricle.
Consequently, regional variations of the wall stress reported here only account for geometric factors; they do not integrate the local changes in pressure.

The evaluation of the wall surface curvature depends on factors such as image resolution and the surface reconstruction process. The spacing between short-axis image slices for cardiac MRI in clinical practice is typically ~5–10 mm. Therefore, a considerable amount of interpolation between image slices has been used, and this may affect the accuracy of the curvature evaluation. The accuracy of the surface curvature also depends on the mesh quality and sampling density of the LV models, since the differential properties of each surface point are computed using additional information from the neighboring vertices. Because of the resolution of the MRIs used and a more intricate shape of the apical region, segmentation of LV contours is often more difficult in the apical region than in the middle and basal regions. This could affect the evaluation of the regional curvedness at the apical region, especially at end systole. An additional limitation is the absence of an age-matched control group.

Several parameters, such as wall thickening and regional circumferential or radius strain, have been developed to quantify regional myocardial function. Since exclusion of trabeculation at end systole is impossible because the trabeculae are compacted with the full thickness of myocardium, the wall thickening may be overestimated. Furthermore, calculation of wall thickening will be affected by the axis rotation during the cardiac cycle. Regional strains were not calculated in the analysis of LV regional curvature, wall stress, and wall thickening. This may affect the accuracy of the curvature evaluation. The coefficients of the paraboloid are then obtained by least-squares solution of an overdetermined system of linear equations:

\[
\begin{align*}
\mathbf{c} & = \mathbf{G} \mathbf{z} \\
\mathbf{c} & = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ \vdots \\ c_n \end{bmatrix} \\
\mathbf{G} & = \begin{bmatrix} 1 & p_1 & q_1 & p_1^2 & q_1^2 & \cdots & p_1^n & q_1^n \\ 1 & p_2 & q_2 & p_2^2 & q_2^2 & \cdots & p_2^n & q_2^n \\ \vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 1 & p_n & q_n & p_n^2 & q_n^2 & \cdots & p_n^n & q_n^n \end{bmatrix}
\end{align*}
\]

where \( n \) is the number of points in the neighborhood of a selected surface point.

It is desired that \( c_1 \)–\( c_6 \) are acquired, such that the divergence of the fitted paraboloid and the data points are minimized, which is similar to minimizing the differences between \( z_n \) (measured value of \( z \)) and \( z'_n \) (calculated value of \( z \)). The minimization problem at this point is then expressed as

\[
E(z) = \sum_{n=1}^{N} G_n(z_n - z'_n)^2 = \sum_{n=1}^{N} G_n(z_n - z'_n)^2 = (A4)
\]

where \( N \) represents the number of points within the fitted paraboloid and \( G \) is a distance weighting function.
Fig. B1. A: LV geometry. B: schematics of 2DSR (cavity radius) and 2DST (wall thickness) in short-axis plane. C: schematics of 2DLR and 2DLT in long-axis plane. D: schematics of 3DR and 3DT accounting for 3-D curvature.

\[ G_i = f \cdot e^{-|r|^2 + \phi_i^2/\alpha^2} \quad i = 1, 2, 3, \ldots, N \quad (A5) \]

where \( f \) and \( \alpha \) are arbitrary constants, which can be adjusted accordingly. The error function \( E \) will have a minimum when

\[ \frac{\partial E}{\partial c_i} = 0 \quad i = 1, 2, 3, 4, 5, 6 \quad (A6) \]

thus yielding the six linear equations required to compute \( c_1-c_6 \).

The first coefficient \( c_1 \) is the \( z \) value of the center point \( x_0 \) on the fitted surface patch, while \( c_2-c_6 \) can be interpreted as the first and second derivatives of \( x \) with respect to \( p \) and \( q \) at \( x_0 \). By acquiring these coefficients, we can then determine the shape descriptor, which is based on the curvedness \( (C) \) presented by Koenderink and Van Doorn (25) as

\[ C = \frac{\kappa_1^2 + \kappa_2^2}{2} \quad (A7) \]

where \( \kappa_1 \) and \( \kappa_2 \) are the maximum and minimum principal curvatures at the vertex of the surface, the values of which are obtained from the roots of

\[ \text{det} \begin{bmatrix} L - kE & M - kF \\ M - kF & N - kG \end{bmatrix} = 0 \quad (A8) \]

where \( \{E, F, G\} \) and \( \{L, M, N\} \) are components of the first and second fundamental forms, respectively.

**APPENDIX B**

Computation of radius and wall thickness using the short- and long-axis approach and the 3-D approach. To enable the calculations of 2DSR and 2DLD, we need to determine the properties of the intersection curve between the quadric surface and the intersecting plane, be it the short- or long-axis plane (Fig. B1). The objective is to find the curvature of the intersection curve, as well as the vector defining the direction, to measure the wall thickness. The wall radius is then the inverse of the curvature. The direction vector will be used to perform a ray-triangle intersection with the epicardial surface mesh to determine the wall thickness.

The calculation is carried out in the local frame of the quadric surface obtained during the patch fitting. As the intersection plane passes through the origin of the quadric surface, the implicit form in the local frame is given by

\[ px + qy + rz = 0 \quad (B1) \]

If we include the quadric equation in the plane equation, we obtain

\[ px + qy + r(ax^2 + bxy + cy^2 + dx + ey) = 0 \quad (B2) \]

The parametric curve that describes the intersection between the quadric and the plane is then given by

\[ \mathbf{r}(\alpha) = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \\ ax^2 + bxy + cy^2 + dx + ey \end{bmatrix} \quad (B3) \]

where \( \alpha \) is the curve parameter. A suitable parameterization is to take \( \alpha = x \). On the basis of differential geometry, the curvature \( \kappa \) of \( \mathbf{r} \) at the origin is

\[ \kappa = \frac{|\mathbf{r}(0) \times \mathbf{r}(0)|}{|\mathbf{r}(0)|^3} \quad (B4) \]

where \( \mathbf{r} \) and \( \mathbf{r}' \) are the first and second derivatives of \( \mathbf{r} \) with respect to the curve parameter, respectively.

To calculate these derivatives, we evaluate the \( y \) component of \( \mathbf{r} \). The derivative of Eq. B2 with respect to \( x \) gives

Fig. B2. Normal (\( \hat{n} \)) at endocardial surface.
At the curvature

Differentiating Eq. B5, again with respect to \( x \), yields

Next, we evaluate the \( z \) component of \( \mathbf{r} \) by differentiating the quadratic equation \( z = ax^2 + bxy + cy^2 + dx + ey \) with respect to \( x \)

If we differentiate Eq. B8 with respect to \( x \), we obtain

At \( \alpha = 0, x = y = 0 \). Using the results in Eqs. B6 and B8, we obtain

where \( m = (-dr - p)(er + q) \).

Furthermore, using the results in Eqs. B7 and B9, we obtain

Using Eqs. B10 and B11, we can then use Eq. B4 to determine the curvature \( \kappa \) of the intersection curve \( \mathbf{r} \). The tangent of the curve \( \mathbf{t} \) at the origin was calculated as follows:

The direction of the wall thickness was then taken as the cross-product of the plane normal \( \hat{n} \) and \( \mathbf{t} \) (in the global coordinate space), as illustrated in Fig. B2.

To calculate the wall thickness, a ray was defined with origin at the point of interest and direction \( \mathbf{I} \). An intersection was then performed between the ray and the epicardial surface. The wall thickness was taken as the distance between the point of interest and the point of ray-surface intersection. However, for the 3DR, method which employs the surface-fitting method, the direction of the ray is the normal of the quadric surface at the origin and is given by

GRANTS

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REFERENCES


